Gyrokinetic Simulations in a Reversed Field Pinch

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Acknowledgements - J. Anderson, V. Tangri, C. Hegna, D. Told, J. Sarff

Outline

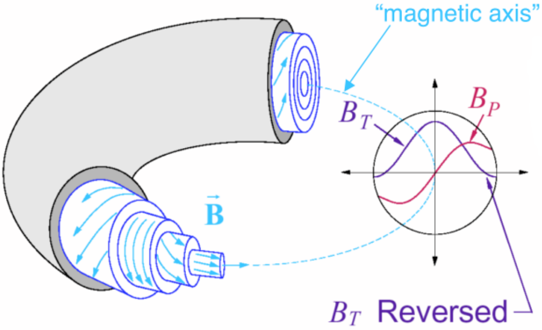
- Introduction
  - The reversed field pinch (RFP)
  - PPCD: a current profile control technique
  - Gyrokinetics and GENE/GYRO
- Generic RFP equilibrium
  - Toroidal Bessel Function Model (GYRO)
  - Linear instability analysis
- Realistic PPCD equilibrium
  - Adjusted Circular Model (GENE)
  - Linear results
- Summary

The Reversed Field Pinch

$B_\phi \sim B_\theta$ is a characteristic feature of RFPs.

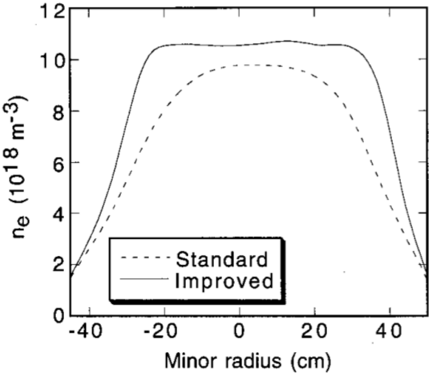
Madison Symmetric Torus:

- Minor radius: 52 cm
- Major radius: 1.54 m



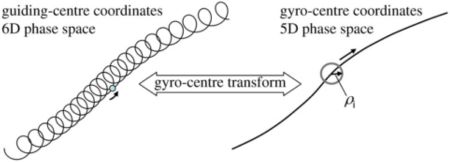
PPCD: increased energy confinement time/gradients

Pulsed poloidal current drive (PPCD) has been responsible for **decreased large scale tearing mode activity** and increased τ_E.



Chapman et al., Phys. Plasmas (2002)

Gyrokinetics and GENE[1]/GYRO[2] code



Garbet et al., 2010

- Kinetic equation reduced to 5D by averaging over the gyrophase angle.
- GENE/GYRO solve the gyrokinetic-Maxwell system of equations

Features of these simulations

- linear
- local (flux-tube)
- initial value

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[1] http://gene.rzg.mpg.de

[2] J. Candy and R.E. Waltz, Journal of Computational Physics (2003)

Modeling of a generic RFP equilibrium

RFP models: the Toroidal Bessel Function Model[3] (TBFM)

Toroidal Bessel Function Model

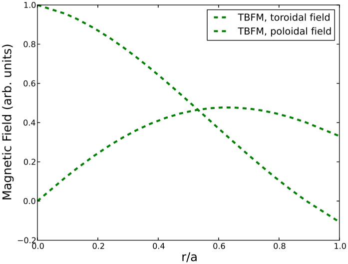
- Solution to Grad-Shafranov equation
- $B_\theta(r) = B_0 J_1\left(\frac{2\Theta r}{a}\right)\frac{1}{1+(r/R_0)cos(\theta)}$
  $B_\phi(r) = B_0 J_0\left(\frac{2\Theta r}{a}\right)\frac{1}{1+(r/R_0)cos(\theta)}$
- Takes $\Theta = \frac{\langle B_\theta \rangle_{wall}}{\langle B_\phi \rangle_{vol}}$ and r as input

TBFM Advantages/Disadvantages:

- Flexible
- More accurate than $s - \alpha$
- Restricted to low Θ

Implemented in GYRO

$\Theta = 1.35$



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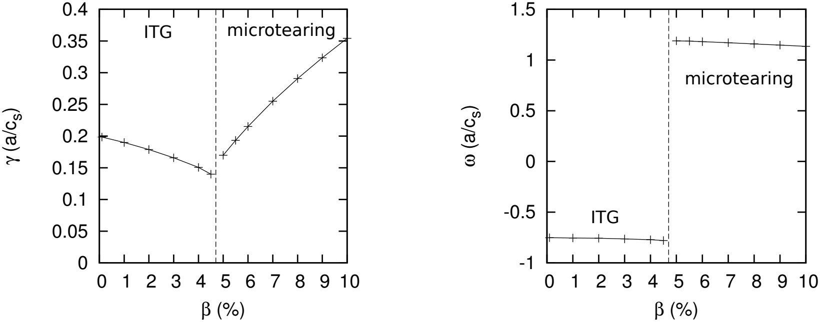
[3] V. Tangri, P.W. Terry, and R.E. Waltz, Phys. Plasmas (2011)

Beta scan reveals transition between ITG and microtearing

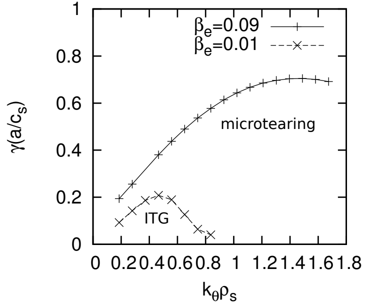
$$k_\theta \rho_s = 0.372, r/a = 0.5, \Theta = 1.35$$
$$a/L_n = 0.58, a/L_{Te} = a/L_{Ti} = 5.0$$

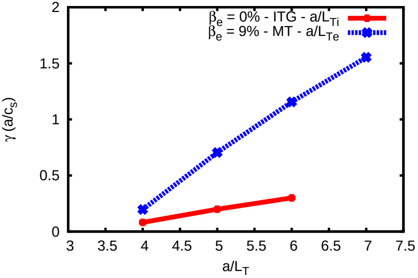


Characteristics of microtearing and ITG

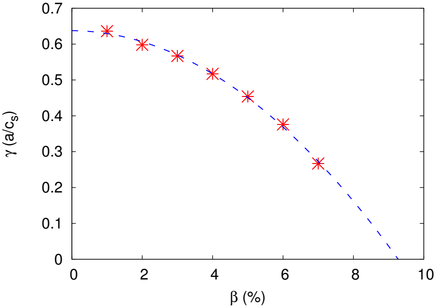
As beta increases, microtearing becomes the dominant mode and its peak shifts to higher values of $k_\theta \rho_s$.

Critical temperature gradients for ITG/MT around $a/L_T \sim 3.5 - 4$





ITG suppressed by finite β



$r_0/a = 0.4$
$q_0 = 0.244$
$a/L_{Te} = 0$
$a/L_n = 0.08$

ITG stabilization occurs near $\beta \approx 9\%$ for these parameters.

Estimate of β for ITG stabilization

RFP geometric terms result in a higher critical β for ITG stabilization than in tokamaks.

$$\omega_{De} = \frac{2cT_e}{eB^3}(\nabla B \times \mathbf{B}) \cdot \mathbf{k} \propto \frac{1}{L_B} \quad k_{\parallel} \sim \frac{z}{qR}$$

tokamak: $L_B \sim R$, $z \sim 1$ RFP: $L_B \sim a$, $z = \frac{1}{\sqrt{1+(\frac{\epsilon}{q})^2}}$

Stabilization Criterion

$$\beta_e \geq \frac{\varepsilon_n \epsilon_t^2 \tau^2}{(1+(\epsilon_t/q_0)^2)q_0^2(\tau+2\varepsilon_n)(\tau+1)+\tau^2\eta_e}$$

$$\eta_e = \frac{L_n}{L_{T_e}},\ \varepsilon_n = \frac{L_n}{L_B},\ \tau = \frac{T_e}{T_i},\ \epsilon_t = r_0/R_0$$

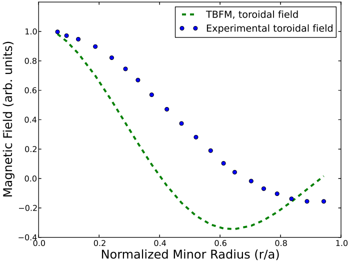
Critical β for ITG suppression scales like $1/L_B$

Modeling a high-Θ experimental PPCD equilibrium

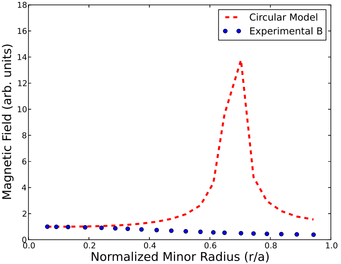
Describing an experimental high Θ discharge

$$\Theta = 2.96$$

Θ too high for TBFM



Gene's tokamak Circular Model has cancellation problem near reversal surface



RFP models: GENE and Circular Model[4]

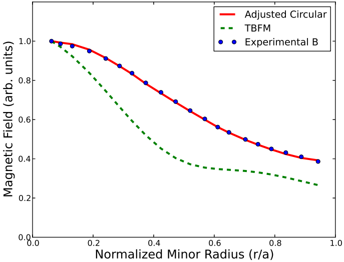
Circular Model

- $\mathbf{B} = \frac{R_0 B_0}{R}(\mathbf{e}_\phi + \mathbf{e}_\theta \frac{r}{R_0 \bar{q}})$
- takes q and shear as inputs

Adjusted Circular Model

- $\mathbf{B} = \frac{R_0 B_0}{R} qf(r)(\mathbf{e}_\phi + \mathbf{e}_\theta \frac{r}{R_0 \bar{q}})$
- Allows for variation of B_ϕ with r.
- $qf = J_0(2\Theta r/a)$ replicates the TBFM.

Implemented in GENE



f is 6th order polynomial

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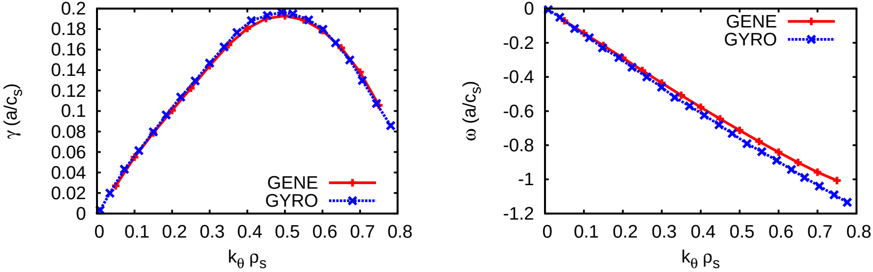
[4] X. Lapillonne, et al. (2009)

GENE/GYRO benchmarking

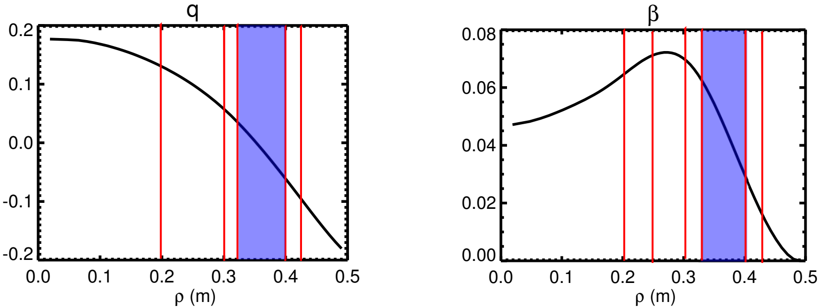
$$\text{TBFM: } qf = J_0(2\Theta r/a) = \frac{RJ_1(2\Theta r/a)q}{r}$$
$$\Theta = 1.35$$



Excellent agreement between codes
for this parameter regime (low Θ).

PPCD: Experimental q and β profiles

$$F = B_\phi(a)/\langle B_\phi \rangle = -1.47,\ \Theta = 2.96$$

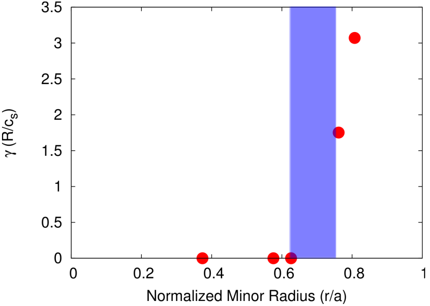


Simulated radial positions.

Region of high shear near reversal surface presents numerical challenges and is excluded from modeling.

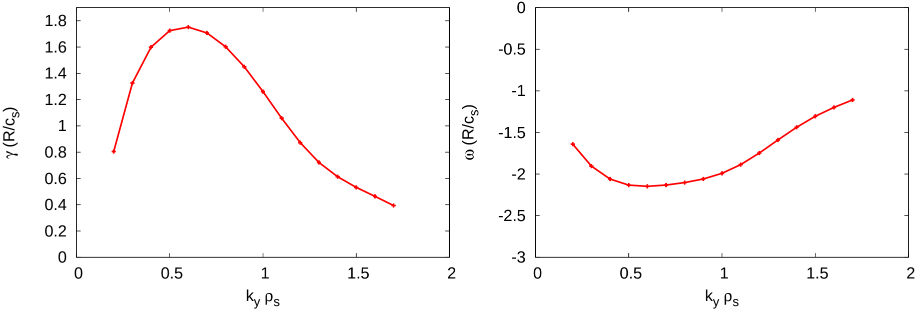
Radial dependence of growth rates shows instability for $r/a \geq 0.75$

At $k_y \rho_s = 0.6$, peak of growth rate spectrum



The instability at $r/a \geq 0.76$ is likely a **density gradient driven trapped electron mode (TEM)**:
frequency in electron direction, ballooning parity,
density gradient dependence

Wavenumber spectrum outside reversal surface, $r/a = 0.76$



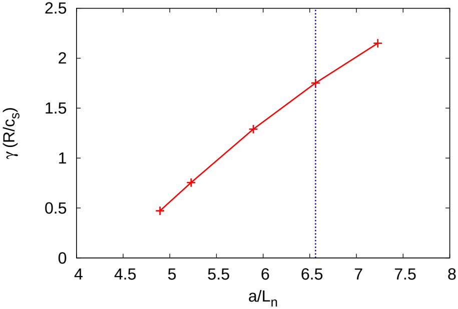
TEM dominant across wide range of wavenumbers.

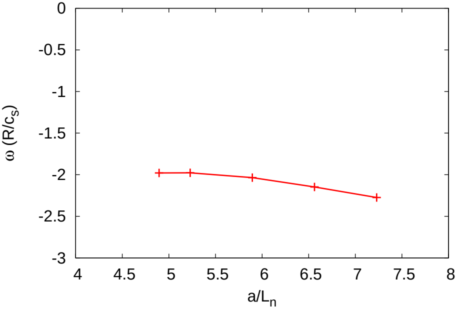
Mode is strongly dependent on density gradients

Critical density gradient around $a/L_n \approx 4.5$

$$\frac{a}{L_n} = \frac{a}{n}\frac{dn}{dr}$$

$$r/a = 0.76,\ k_y\rho_s = 0.6$$





a/L_n is the most important gradient for this mode.

Open questions and future work

- How does the instability spectrum various across different discharges? What are the dominant instabilities?
- How sensitive are the growth rates to various parameters like a/L_n, a/L_T, $\hat{s}$, β?
- Are there new effects that arise when taking into account nonlinear physics?

Summary

- Two different RFP equilibrium models were discussed: **TBFM and Adjusted Circular Model**
- Case 1: generic RFP equilibrium ($\Theta = 1.35$)
  → unstable to ITG (at low β) and MT (at high β)
  → ITG stabilized at higher β relative to tokamak
- Case 2: realistic PPCD equilibrium ($\Theta = 2.96$)
  → unstable to density-gradient driven TEM
  → Instabilities are strong for $r/a \geq 0.76$
  → This PPCD case has relatively low β, gradients - other regimes may occur in different discharges