UW-CPTC 04-9R1, June 30, 2005

Paleoclassical electron heat transport

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Abstract. It has been hypothesized that radial electron heat transport in low collisionality, current-carrying magnetically-confined toroidal plasmas results from paleoclassical Coulomb collision processes (parallel electron heat conduction and magnetic field diffusion). In such plasmas the electron temperature equilibrates along magnetic field lines a long length L, which is the minimum of the electron collision length and a maximum effective half length of helical field lines. Diffusing field lines carry this equilibrated electron temperature with them and thus induce a radial electron heat diffusivity $M \simeq L/(\pi R_0 q) \sim 10 >> 1$ times the resistivity-induced magnetic field diffusivity $\eta/\mu_0 \simeq \nu_e(c/\omega_p)^2$. Interpretations of many features of "anomalous" electron heat transport provided by the paleoclassical model are discussed: magnitude and radial profile of electron heat diffusivity (in tokamaks, STs, and RFPs), Alcator scaling in high density plasmas, electron heat transport barriers around low order rational surfaces and near a separatrix, and a natural heat pinch (or minimum temperature gradient) electron heat flux form. Also, the context (relative to other transport models), regime of applicability, and suggestions for experimental tests of the paleoclassical model are discussed.

PACS numbers: 52.25.Fi, 52.35.Vd, 52.55.Dy, 52.55.Fa

1. Introduction: paleoclassical physical mechanism

The fastest (earliest) and most primitive ("paleo"), dominant Coulomb-collision-induced transport processes in magnetically-confined plasmas occur on the electron collision time scale $1/\nu_e$ and are referred to as paleoclassical [1,2]: parallel electron heat conduction and magnetic field diffusion. On this time scale, the electron distribution is Maxwellianized and electron heat conduction equilibrates the electron temperature T_e over long distances parallel to the magnetic field $\mathbf{B}$ — up to the electron collision length $\lambda_e \equiv v_{Te}/\nu_e$ in which $v_{Te} \equiv (2T_e/m_e)^{1/2}$. Magnetic field diffusion is induced by the plasma resistivity η. It causes magnetic flux (bundles of field lines) to diffuse perpendicular to $\mathbf{B}$ with a diffusion coefficient $D_\eta \simeq \eta_0/\mu_0 \equiv \nu_e(c/\omega_p)^2 \sim (\Delta x)^2/\Delta t$, which implies a diffusive radial step $\Delta x \simeq \delta_e \equiv c/\omega_p$ [electromagnetic (em) skin depth] in a collision time $\Delta t \simeq 1/\nu_e$.

Electron gyromotion about magnetic field lines causes the electron guiding center to be identified with the small amount of poloidal magnetic flux traversed by the radial component of the electron gyromotion. However, since in current-carrying toroidal plasmas the poloidal flux (bundles of field lines) diffuses radially due to D_η, it was hypothesized [1,2] that the electron guiding center position becomes a radially diffusing "stochastic variable." To account for this effect a spatial Fokker-Planck operator [3,4] was added [1,2] to the usual drift-kinetic equation — see (34), (35). If λ_e is longer than the length of a helical field line on a $q_* \equiv m/n$ rational surface or the effective parallel length of diffusing rational field lines [for $n \leq n_{\max} \sim 10$ — see (41)], the parallel equilibration length L is reduced to these lengths — see (46). The effect of the T_e equilibration over a long length L along radially diffusing helical rational field lines that are longer than the poloidal periodicity half length ($\sim \pi R_0 q$) is that the effective electron heat diffusivity is a multiple $M \sim L/(\pi R_0 q) \sim 10$ of the magnetic field diffusivity D_η — see (40), (45), (48), and (52).

The paleoclassical model for radial electron heat transport was first introduced using a sheared slab magnetic field model [1]. Thereafter, it was developed in some detail for an axisymmetric magnetic magnetic field geometry [2]. The key results of those papers are elucidated and summarized in the first four sections of this paper: 2. Magnetic field geometry, 3. Magnetic flux, field line diffusion, 4. Paleoclassical kinetics, analysis, and 5. Paleoclassical radial electron heat transport. In Section 5 a number of new encouraging comparisons of the paleoclassical model for radial electron heat transport with experimental data [for $T_e \lesssim 1$ keV$\times B(\mathrm{T})^{2/3} a(\mathrm{m})^{1/2}$] are discussed. Section 6 discusses the context, range of applicability and limitations of the present paleoclassical model, and suggests a number of local transport and fundamental tests of it. The final section provides a brief summary of this paper.

2. Magnetic field geometry

The paleoclassical model will be summarized here using a full axisymmetric magnetic field model for arbitrary aspect ratio ($A \equiv R_0/r \equiv 1/\epsilon$ where R_0, r are the major, minor radii of a flux surface) to facilitate application of the theory to most types of axisymmetric toroidal plasmas — large aspect ratio tokamaks ($A >> 1$) and regions of spherical tokamaks (STs, $A \gtrsim 1$), spheromaks, and reversed field pinches (RFPs) where $\epsilon^2, B_p^2/B_t^2 << 1$. Approximate results for large aspect ratio tokamaks are indicated at the end of many equations after an approximate equality ($\simeq$).

Paleoclassical transport is concerned with diffusion of magnetic flux (bundles of magnetic field lines). Since for axisymmetric toroidal plasmas with $\epsilon^2, B_p^2/B_t^2 << 1$ the toroidal magnetic flux ψ_t is less mobile than the poloidal magnetic flux ψ [5–7], diffusion of the poloidal flux surfaces (and field lines) will be determined relative to ψ_t and hence to a dimensionless, cylindrical-type radial flux surface label ρ: $\rho \equiv [\psi_t/\psi_t(a)]^{1/2} \simeq r/a$, $\psi_t(\rho,t) \equiv (1/2\pi)\iint d\mathbf{S}(\zeta)\cdot\mathbf{B}_t \simeq r^2 B_0/2$. The appropriate magnetic field model [5–7] has toroidal (t) and poloidal (p) components: $\mathbf{B} = \mathbf{B}_t + \mathbf{B}_p = I\,\boldsymbol{\nabla}\zeta + \boldsymbol{\nabla}\zeta\times\boldsymbol{\nabla}\psi = \boldsymbol{\nabla}\psi\times\boldsymbol{\nabla}(q\,\theta - \zeta)$. As usual, $I = I(\rho,t) \equiv RB_t \simeq B_0 R_0$. Also, ζ is the toroidal angle and the poloidal flux is $\psi(\rho,t) \equiv (1/2\pi)\iint d\mathbf{S}(\theta)\cdot\mathbf{B}_p$, $\partial\psi/\partial\rho \simeq aR_0 B_p$. Further, θ is the straight-field-line (in the ψ = constant plane) poloidal angle and q is the winding number or pitch ("safety factor" for kink stability) of field lines on a flux surface:

$q(\rho,t) \equiv (\partial\psi_t/\partial\rho)/(\partial\psi/\partial\rho) =$ # toroidal transits/# poloidal transits $\simeq r\,B_t/R_0 B_p$. For an axisymmetric magnetic field $q(\rho,t) = q(\psi,t)$, $\mathbf{B}\cdot\boldsymbol{\nabla}\theta = I/qR^2 \simeq B_t/R_0 q = B_p/r$.

The Jacobian for transforming from the original Eulerian coordinates to the curvilinear set $u^i \equiv (\rho,\theta,\zeta)$ is $\sqrt{g} \equiv 1/\boldsymbol{\nabla}\rho\cdot\boldsymbol{\nabla}\theta\times\boldsymbol{\nabla}\zeta = (\partial\psi/\partial\rho)/\mathbf{B}\cdot\boldsymbol{\nabla}\theta \simeq raR_0$. The radial differential of the volume is $V' \equiv \partial V(\rho,t)/\partial\rho = 2\pi\int_{-\pi}^{\pi}\sqrt{g}\,d\theta \simeq a(2\pi r)(2\pi R_0)$. The average of an axisymmetric ($\partial f/\partial\zeta = 0$) scalar function $f(\mathbf{x},t)$ over a flux surface is $\langle f(\mathbf{x},t)\rangle = (2\pi/V')\int_{-\pi}^{\pi}\sqrt{g}\,d\theta\;\; f(\mathbf{x},t)$. The flux-surface-average is an annihilator for the parallel gradient operator: $\langle\mathbf{B}\cdot\boldsymbol{\nabla}f\rangle = 0$, for any function $f(\mathbf{x},t)$ that is periodic in both θ and ζ. For a similarly periodic vector field $\mathbf{A}(\mathbf{x},t)$, the flux-surface-average of its divergence, defined by $\boldsymbol{\nabla}\cdot\mathbf{A} \equiv \sum_i (1/\sqrt{g})(\partial/\partial u^i)(\sqrt{g}\mathbf{A}\cdot\boldsymbol{\nabla}u^i)$, becomes $\langle\boldsymbol{\nabla}\cdot\mathbf{A}\rangle = (\partial/\partial V)\langle\mathbf{A}\cdot\boldsymbol{\nabla}V\rangle$.

Flux surfaces are rational or irrational depending on whether q is the ratio of integers (m,n):

$$q(\rho,t)\;\begin{cases} = m/n, & \text{rational surface,}\\ \neq m/n, & \text{irrational surface.}\end{cases} \tag{1}$$

The irrational surfaces form a dense set while the finite m,n ($\leq n_{\max}$) rational surfaces are a set of measure zero and radially isolated from each other. Rational surfaces are of interest here because their (helical) magnetic field lines close on themselves after m toroidal (or n poloidal) transits.

The differential length $d\ell$ along magnetic field lines obtained from the poloidal ($\boldsymbol{\nabla}\theta$) projection of the field line equation $d\mathbf{x}/d\ell = \mathbf{B}/B$ is $d\ell = (B/\mathbf{B}\cdot\boldsymbol{\nabla}\theta)\,d\theta \simeq R_0 q\,d\theta$. The half length ℓ_* of a closed helical field line on a $q_* \equiv q(\rho_*) \equiv m/n$ rational surface is [2]:

$$\ell_* \equiv \frac{1}{2}\int_{-n\pi}^{n\pi}\frac{B\,d\theta}{\mathbf{B}\cdot\boldsymbol{\nabla}\theta} = \pi\bar{R}q_* n, \quad q_* \text{ line length,} \tag{2}$$

$$\bar{R} \equiv \frac{1}{2\pi q_*\partial\psi/\partial\rho}\int_{-\pi}^{\pi}\sqrt{g}\,d\theta\,B = \frac{\langle B\rangle\;V'}{4\pi^2 q_*\partial\psi/\partial\rho} \simeq R_0. \tag{3}$$

While helical field lines on medium order rational surfaces with $n \sim 10 >> 1$ are long ($>> \pi\bar{R}q_*$), those with low n ($\equiv n^\circ = 1,2$) are short ($\ell_{n^\circ} \equiv \pi\bar{R}q_* n^\circ \sim \pi\bar{R}q_*$).

Radial distances between medium order rational surfaces can be estimated using a Taylor series expansion of $q(\rho,t)$ about a rational surface at $\rho = \rho_*$:

$$q(\rho,t) \simeq q_* + x\,q' + \mathcal{O}(x^2), \quad q' \equiv |\partial q/\partial\rho|_{\rho_*}, \tag{4}$$

$$q_* \equiv q(\rho_*) = m/n, \tag{5}$$

$$x \equiv \rho - \rho_*. \tag{6}$$

The distance between rational surfaces with $m \pm 1$ but the same n is obtained from $1/n = q - q_* \simeq xq'$:

$$\Delta \simeq 1/nq', \quad \text{same } n \text{ rational surface spacing.} \tag{7}$$

Defining $q(\rho_{\max}) = m_{\max}/n_{\max}$ and expanding $q(\rho) = (m_{\max}n+1)/n_{\max}n$ about $\rho = \rho_{\max}$ yields the distance between a $q_* \equiv m/n$ rational surface and the nearest $n \leq n_{\max}$ rational surface:

$$\delta x(n) \equiv \rho_* - \rho_{\max} \simeq \frac{1}{n_{\max}n\,q'}\,, \tag{8}$$

which is the minimum spacing for $q' \neq 0$, $n \leq n_{\max}$. At a minimum in q where q' vanishes, one obtains

$$\delta x_{\min}(n) \equiv \rho_* - \rho_{\max} \simeq \left(\frac{2}{n_{\max} n\, q''}\right)^{1/2}, \tag{9}$$

in which $q'' \equiv \partial^2 q/\partial\rho^2|_{\rho_*}$. For $n_{\max} \gtrsim 10$, $q' \sim 1$, and $q'' \sim 1$, all of these distances are small fractions of the minor radius: $\Delta \sim 1/n_{\max} << 1$ for $n \sim n_{\max}$, and $\delta x(n) \lesssim 1/n_{\max} << 1$, $\delta x_{\min}(n) \lesssim 1/\sqrt{n_{\max}} < 1$.

3. Magnetic flux, field line diffusion

Evolution equations for the toroidal flux ψ_t and the poloidal flux ψ obtained from Faraday's law ($\partial\mathbf{B}/\partial t = -\boldsymbol{\nabla}\times\mathbf{E}$) are [2,5–7]:

$$\frac{d\psi_t}{dt} \equiv \left.\frac{\partial\psi_t}{\partial t}\right|_{\mathbf{x}} + \langle\mathbf{u}_g\cdot\boldsymbol{\nabla}\psi_t\rangle = 0, \tag{10}$$

$$\frac{d\psi}{dt} \equiv \left.\frac{\partial\psi}{\partial t}\right|_{\mathbf{x}} + \langle\mathbf{u}_g\cdot\boldsymbol{\nabla}\psi\rangle = \frac{\langle\mathbf{E}\cdot\mathbf{B}\rangle}{\langle\mathbf{B}\cdot\boldsymbol{\nabla}\zeta\rangle} - \frac{\partial\Psi}{\partial t}, \tag{11}$$

$$\langle\mathbf{u}_g\cdot\boldsymbol{\nabla}\psi_t\rangle \equiv q\,\frac{\langle\mathbf{E}\cdot\mathbf{B}_p\rangle}{\langle\mathbf{B}\cdot\boldsymbol{\nabla}\zeta\rangle}, \qquad \text{“grid velocity.”} \tag{12}$$

The toroidal flux ψ_t is advected radially by the grid velocity $\mathbf{u}_g$ induced by the poloidal electric field, but conserved in a Lagrangian frame. In (11) $\partial\Psi/\partial t \equiv V^{\zeta}_{\rm loop}(t)/2\pi$ is the (positive) constant of a spatial integration. It represents the toroidal loop voltage induced by the rate of change of the magnetic flux in the central solenoid of a tokamak. The poloidal flux ψ and hence poloidal magnetic field lines move relative to ψ_t [compare (10) and (11)] because of departures from ideal MHD (i.e., a nonzero parallel electric field $\langle\mathbf{E}\cdot\mathbf{B}\rangle$) or a temporally changing magnetic flux in the central solenoid (i.e., $\partial\Psi/\partial t \neq 0$).

A parallel Ohm's law for $\langle\mathbf{E}\cdot\mathbf{B}\rangle$ is obtained [2] from the flux-surface-average of the parallel ($\mathbf{B}\,\cdot$) component of the electron momentum equation including inertial and viscosity effects:

$$\frac{\langle\mathbf{E}\cdot\mathbf{B}\rangle}{\langle\mathbf{B}\cdot\boldsymbol{\nabla}\zeta\rangle} = \left(\frac{\eta_\parallel^{\rm nc}}{\mu_0} + \delta_e^2\frac{d}{dt}\right)\frac{\langle\mu_0\mathbf{J}\cdot\mathbf{B}\rangle}{\langle\mathbf{B}\cdot\boldsymbol{\nabla}\zeta\rangle} + \frac{\mu_e}{\nu_e}\eta_0\frac{1}{\langle R^{-2}\rangle}\frac{dP}{d\psi}. \tag{13}$$

Here, the terms on the right indicate: magnetic flux diffusion [see (22) below] induced by the neoclassical parallel resistivity, electron inertia, and the neoclassical bootstrap current. The (neoclassical) parallel electrical resistivity (neglecting poloidal electron heat flow effects) is

$$\boxed{\frac{\eta_\parallel^{\rm nc}}{\eta_0} \simeq \frac{\eta_\parallel^{\rm Sp}}{\eta_0} + \frac{\mu_e}{\nu_e}, \quad \text{neoclassical resistivity,}} \tag{14}$$

in which $\eta_\parallel^{\rm Sp}$ is the classical parallel resistivity:

$$\frac{\eta_\parallel^{\rm Sp}}{\eta_0} \simeq \frac{\sqrt{2}+Z}{\sqrt{2}+13Z/4}, \quad \text{Spitzer resistivity.} \tag{15}$$

The reference ($\perp$) resistivity η_0 and electron viscous drag frequency μ_e adapted from [6,7] are

$$\boxed{\frac{\eta_0}{\mu_0} \equiv \frac{m_e \nu_e}{n_e e^2 \mu_0} = \nu_e \delta_e^2 \simeq \frac{1.4 \times 10^3\, Z}{[T_e(\mathrm{eV})]^{3/2}} \left(\frac{\ln\Lambda}{17}\right) \frac{\mathrm{m}^2}{\mathrm{s}},} \tag{16}$$

$$\frac{\mu_e}{\nu_e} \simeq \frac{Z + \sqrt{2} - \ln(1+\sqrt{2})}{Z\,(1 + \nu_{*e}^{1/2} + \nu_{*e})} \frac{f_t}{f_c} \overset{\nu_{*e}=0}{\underset{Z=1}{\Longrightarrow}} 1.5\, \frac{f_t}{f_c}. \tag{17}$$

Here, Z ($\rightarrow Z_{\mathrm{eff}} \equiv \sum_i n_i Z_i^2/n_e$ for multiple ion species) is the (effective) ion charge, and f_c is the flow-weighted fraction of circulating particles [7] with Padè approximation [8]

$$f_c \simeq \frac{(1-\epsilon^2)^{-1/2}\,(1-\epsilon)^2}{1 + 1.46\epsilon^{1/2} + 0.2\epsilon} \simeq 1 - 1.46\epsilon^{1/2} \tag{18}$$

Further, the fraction of trapped particles is $f_t \equiv 1 - f_c$ and the electron collisionality parameter is defined by

$$\nu_{*e} \equiv \frac{\nu_e}{\epsilon^{3/2}(v_{Te}/\bar{R}q)} = \frac{\bar{R}q}{\epsilon^{3/2}\lambda_e}, \tag{19}$$

in which the electron collision length λ_e is given by

$$\lambda_e \equiv \frac{v_{Te}}{\nu_e} \simeq 1.2 \times 10^{16}\, \frac{[T_e(\mathrm{eV})]^2}{n_e\, Z} \left(\frac{17}{\ln\Lambda}\right)\ \mathrm{m}. \tag{20}$$

The $\eta_\parallel^{\mathrm{nc}}$ in (14) ranges [2] from being equal (for $\mu_e/\nu_e << 1$) to twice (for $\mu_e/\nu_e >> 1$) the most precise neoclassical resistivity results [6,7,9].

From Ampere's law, $\mu_0 \mathbf{J} \equiv \boldsymbol{\nabla}\times\mathbf{B} = (\partial I/\partial\psi)\boldsymbol{\nabla}\psi\times\boldsymbol{\nabla}\zeta + \boldsymbol{\nabla}\zeta\,\Delta^*\psi$, in which the usual magnetic differential operator is $\Delta^*\psi \equiv (1/|\boldsymbol{\nabla}\zeta|^2)\boldsymbol{\nabla}\cdot|\boldsymbol{\nabla}\zeta|^2\boldsymbol{\nabla}\psi$. Dotting this $\mu_0\mathbf{J}$ with $\mathbf{B}$, flux surface averaging, and using $|\boldsymbol{\nabla}\zeta|^2 = R^{-2}$ yields [2,6]

$$\Delta^+\psi \equiv \frac{\langle \mu_0 \mathbf{J}\cdot\mathbf{B}\rangle}{\langle \mathbf{B}\cdot\boldsymbol{\nabla}\zeta\rangle} = \frac{I}{\langle R^{-2}\rangle V'}\frac{\partial}{\partial\rho}\left[\left\langle \frac{|\boldsymbol{\nabla}\rho|^2}{R^2}\right\rangle \frac{V'}{I}\frac{\partial\psi}{\partial\rho}\right] \simeq \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial\psi}{\partial r}. \tag{21}$$

Substituting the Ohm's-law-determined $\langle \mathbf{E}\cdot\mathbf{B}\rangle$ in (13) using $\langle \mathbf{J}\cdot\mathbf{B}\rangle$ from (21) into the poloidal flux evolution equation (11), one obtains a diffusion-type (at least for $\delta_e^2\Delta^+ << 1$) equation for ψ [2]:

$$\frac{d}{dt}\left(1 - \delta_e^2\Delta^+\right)\psi = D_\eta\,\Delta^+\psi - S_\psi, \tag{22}$$

$$\boxed{D_\eta \equiv \frac{\eta_\parallel^{\mathrm{nc}}}{\mu_0}, \quad \text{magnetic field diffusivity.}} \tag{23}$$

Sources of poloidal flux in $S_\psi \equiv \partial\Psi/\partial t - (\mu_e/\nu_e)\eta_0(1/\langle R^{-2}\rangle)dP/d\psi$ arise from the "current-drive" effects due to a changing flux in the central solenoid ($\partial\Psi/\partial t \equiv V_{\mathrm{loop}}^{\zeta}/2\pi$) and the bootstrap current. An extra, noninductive current density source $\mathbf{J}_S$ would add a term $\eta_\parallel^{\mathrm{nc}}\langle \mathbf{J}_S\cdot\mathbf{B}\rangle/I\langle R^{-2}\rangle$ to the source S_ψ [2]. The flux diffusion and paleoclassical transport processes discussed below will occur whenever $D_\eta\,\Delta^+\psi \propto \eta_\parallel^{\mathrm{nc}}\langle \mathbf{J}\cdot\mathbf{B}\rangle \neq 0$, i.e., whenever flux-surface-averaged parallel current flows in the resistive plasma. Thus, they will occur even if a noninductive current source causes the toroidal loop voltage to vanish since then the inductive source $\partial\Psi/\partial t$ in S_ψ is just replaced by the bootstrap current and/or noninductive current source $\langle \mathbf{J}_S \cdot \mathbf{B}\rangle$.

In equilibrium in the Lagrangian frame, $d/dt \to 0$ and the equation for the stationary poloidal flux ψ_0 becomes $0 = D_\eta \Delta^+ \psi_0 - S_\psi$. Thus, in equilibrium the diffusion of ψ (poloidal flux, field lines) is balanced by the source S_ψ of poloidal flux; the Poynting flux represented by $\partial\Psi/\partial t$ brings poloidal field lines into the plasma and the magnetic field diffusivity D_η diffuses them out of the plasma — even for a stationary poloidal magnetic field $\mathbf{B}_p$, which will be assumed henceforth. The combination of the source S_ψ and the diffusion of the poloidal magnetic flux ψ induced by η causes the poloidal magnetic field to be a driven-dissipative system on the resistive ("skin") time scale $\tau_\eta \sim a^2/\,6D_\eta = \mu_0 a^2/\,6\eta_\parallel^{\rm nc}$ in current-carrying toroidal plasmas.

To determine the properties of the small bundle of poloidal magnetic flux $\delta\psi(x,t)$ traversed by the electron gyroorbit, one substitutes an Ansatz of $\psi \to \psi_0 + \delta\psi$ into (22) to obtain (for $x^2 << 1$) $(\partial/\partial t + \bar{u}_g\,\partial/\partial x)(1 - \bar{\delta}_e^2\,\partial^2/\partial x^2)\,\delta\psi \simeq \bar{\nu}_e \bar{\delta}_e^2\,\partial^2\delta\psi/\partial x^2$, in which the following normalized variables have been defined:

$$\bar{u}_g \equiv \langle \mathbf{u}_g \cdot \boldsymbol{\nabla}\rho\rangle, \quad \bar{\delta}_e \equiv \frac{\delta_e}{\bar{a}}, \tag{24}$$

$$\frac{1}{\bar{a}^2} \equiv \frac{1}{\langle R^{-2}\rangle}\left\langle \frac{|\boldsymbol{\nabla}\rho|^2}{R^2}\right\rangle \simeq \frac{1}{a^2}, \tag{25}$$

$$\bar{D}_\eta \equiv \frac{D_\eta}{\bar{a}^2}, \quad \bar{\nu}_e \equiv \nu_e \frac{\eta_\parallel^{\rm nc}}{\eta_0}. \tag{26}$$

For $|x| < \bar{\delta}_e$ (or $k_x^2\bar{\delta}_e^2 > 1$), the $\delta\psi$ solution is spatially constant [2]; hence, it produces no field lines or diffusion of them in this region (i.e., $\delta\mathbf{B}_p \equiv \boldsymbol{\nabla}\zeta \times \boldsymbol{\nabla}\delta\psi = \mathbf{0}$ there).

For radial scale lengths longer than the em skin depth δ_e (for which $k_x^2\bar{\delta}_e^2 << 1$ so that $\delta_e^2\Delta^+ << 1$ can be neglected), the evolution equation for $\delta\psi$ becomes a simple diffusion equation:

$$\frac{d\,\delta\psi}{dt} \equiv \left(\frac{\partial}{\partial t} + \bar{u}_g \frac{\partial}{\partial x}\right)\delta\psi = \bar{D}_\eta\,\frac{\partial^2\delta\psi}{\partial x^2}. \tag{27}$$

Its (Green function) solution for a small bundle of flux initially located at $x = x_0$ $(> \bar{\delta}_e)$, which is represented by $\delta\psi(x, t = 0) = \delta\psi_0\,\delta(x - x_0)$, is

$$\delta\psi(x,t) = \delta\psi_0\,\frac{e^{-(x-x_0-\bar{u}_g t)^2/4\bar{D}_\eta t}}{(4\pi\bar{D}_\eta t)^{1/2}}. \tag{28}$$

Note that $\delta\psi$ indicates a temporally evolving probability distribution for the radial location of the $\delta\psi$ bundle of poloidal flux (field lines) that was initially at $x = x_0$. The mean displacement and radial spread (variance) of this bundle of magnetic flux are

$$\frac{\int_{-\infty}^{\infty} dx\,(x - x_0)\,\delta\psi}{\int_{-\infty}^{\infty} dx\,\delta\psi} = \bar{u}_g\,t, \tag{29}$$

$$\frac{\int_{-\infty}^{\infty} dx\,(x - x_0)^2\,\delta\psi}{\int_{-\infty}^{\infty} dx\,\delta\psi} = 2\bar{D}_\eta t = 2\,\bar{\nu}_e t\,\bar{\delta}_e^2. \tag{30}$$

As indicated, the average radial displacement and spread (variance) of this small amount of poloidal flux (bundle of field lines) grow linearly with time. Note that this magnetic flux (field line) advection and diffusion process occurs even when the poloidal magnetic field $\mathbf{B}_p$ is in stationary equilibrium (i.e., $d\psi/dt = 0$), which is being assumed here.

In the next section a Fokker-Planck model will be used to include effects of radial advection and diffusion of field lines in a kinetic analysis. Relevant Fokker-Planck coefficients deduced from (29), (30) are (for $x^2 > \bar{\delta}_e^2$)

$$\frac{\langle \Delta x \rangle}{\Delta t} = \bar{u}_g, \quad \frac{\langle (\Delta x)^2 \rangle}{2\,\Delta t} = \bar{D}_\eta. \tag{31}$$

The Fokker-Planck coefficients can be written in a general vectorial form in terms of the covariant base vector in the "radial" direction $\mathbf{e}_\rho \equiv \partial \mathbf{x}/\partial \rho = \sqrt{g}\,\boldsymbol{\nabla}\theta\boldsymbol{\times}\boldsymbol{\nabla}\zeta$, for which $\mathbf{e}_\rho\boldsymbol{\cdot}\boldsymbol{\nabla}\rho = 1$:

$$\frac{\langle \Delta \mathbf{x} \rangle}{\Delta t} \equiv \frac{\langle \Delta x \rangle}{\Delta t}\mathbf{e}_\rho, \quad \frac{\langle \Delta \mathbf{x} \Delta \mathbf{x} \rangle}{\Delta t} \equiv \frac{\langle (\Delta x)^2 \rangle}{\Delta t}\mathbf{e}_\rho \mathbf{e}_\rho. \tag{32}$$

To consider diffusion of helical flux (field lines) in the vicinity of a rational surface at $\rho = \rho_*$ where $q_* \equiv q(\rho_*) = m/n$, one uses a local helical coordinate system $u^i \equiv (\rho, \theta, \alpha)$ with helical angle [10] $\alpha \equiv \zeta - q_*\theta = \zeta - (m/n)\,\theta$. Since $\boldsymbol{\nabla}\theta\boldsymbol{\times}\boldsymbol{\nabla}q_*\theta = \mathbf{0}$, the Jacobian $\sqrt{g} \equiv (\boldsymbol{\nabla}\rho\boldsymbol{\cdot}\boldsymbol{\nabla}\theta\boldsymbol{\times}\boldsymbol{\nabla}\alpha)^{-1}$ is the same as before. Thus, one writes $\mathbf{B}$ locally as [2] $\mathbf{B} = \boldsymbol{\nabla}\alpha\boldsymbol{\times}\boldsymbol{\nabla}\psi + \boldsymbol{\nabla}\psi_*\boldsymbol{\times}\boldsymbol{\nabla}\theta \equiv \mathbf{B}_h + \mathbf{B}_*$, in which the helical flux ψ_* is defined by

$$\partial\psi_*/\partial\rho = (q - q_*)\,\partial\psi/\partial\rho. \tag{33}$$

Near a rational surface using $q(\rho) \simeq q_* + xq'$ one obtains $\psi_{*0} \simeq (x^2/2)\,q'\,\psi'$. Integrating (33) over ρ near $\rho = \rho_*$, taking its total time derivative, and using $d\psi_t/dt = 0$ from (10), one obtains [2] (again neglecting em skin depth effects for $x^2 > \bar{\delta}_e^2$) $d\psi_*/dt = -\,q_*\,d\psi/dt$. Thus, the helical flux ψ_* diffuses like the poloidal flux ψ does. Also, writing $\psi_* \to \psi_{*0} + \delta\psi_*$, one finds [2] that $\delta\psi_*$ obeys the same diffusion-type equation as $\delta\psi$ does, i.e., (27). Hence helical flux (field lines) in the vicinity of rational surfaces also advects and diffuses with the Fokker-Planck coefficients given in (31), (32).

4. Paleoclassical kinetics, analysis

In drift-kinetic theory the magnetic flux surfaces and field lines are usually assumed to be fixed in space; however, as discussed in the preceding section, when $\eta\,\langle \mathbf{J}\boldsymbol{\cdot}\mathbf{B}\rangle \neq 0$ the poloidal flux, field lines diffuse radially. The question then becomes: how do the electrons, and in particular their guiding centers, respond to the diffusing poloidal magnetic flux, field lines?

Because of the axisymmetry, the canonical momentum in the toroidal direction (with covariant base vector $\mathbf{e}_\zeta \equiv R^2\boldsymbol{\nabla}\zeta$) is a constant of the "collisionless," "dissipationless" electron motion: $p_\zeta \equiv \mathbf{e}_\zeta\boldsymbol{\cdot}(m_e\mathbf{v} + q_e\mathbf{A}) = m_e v_\zeta + q_e A_\zeta =$ constant. For the axisymmetric toroidal magnetic field being used here, $A_\zeta = \mathbf{e}_\zeta\boldsymbol{\cdot}(-\psi\boldsymbol{\nabla}\zeta) = -\psi$. Thus, averaging over the electron gyroperiod ($2\pi/\omega_{ce}$ with $\omega_{ce} \equiv q_e B/m_e$), the dissipationless constant of the electron guiding center motion can be written in the usual form $-p_\zeta/q_e = \psi - v_\parallel R B_t/\omega_{ce}$. However, in driven-dissipative, current-carrying toroidal plasma confinement systems, p_ζ is not a constant, at least on the transport (or resistive diffusion time τ_η) time scale.

Neoclassical transport [6,7] results from the stochastic diffusion of the $v_\parallel$ term in p_ζ due to the diffusion in velocity space induced by Coulomb collisions, which is represented in the drift-kinetic equation by the Fokker-Planck collision operator. The key hypothesis of the paleoclassical model [1,2] is that, as a corollary, the spatial advection and diffusion of small poloidal magnetic flux bundles (i.e., those traversed by the electron gyroorbit) carry the electron guiding centers with them. Thus, they cause p_ζ and hence the electron guiding center position to become a stochastic, diffusing variable [2]. That is, the Fokker-Planck spatial advection and diffusion coefficients in (31), (32) represent not just the motion of small poloidal flux bundles but also the motion of the electron guiding centers, at least for $x^2 > \bar{\delta}_e^2$.

The relevant electron kinetic equation is the gyro-averaged one, which is called the drift-kinetic equation [6]. Adding the Fokker-Planck-type effects [3,4] of the radial diffusion of the electron guiding centers induced by the poloidal magnetic flux (field line) diffusion, the magnetic-field-diffusion-Modified Drift-Kinetic Equation (MDKE) is

$$\boxed{\frac{\partial f}{\partial t} + \mathbf{v}_g \boldsymbol{\cdot} \boldsymbol{\nabla} f + \dot{\varepsilon}\, \frac{\partial f}{\partial \varepsilon} = \mathcal{C}\{f\} + \mathcal{D}\{f\}.} \tag{34}$$

Here, $f = f(\mathbf{x}_g, \varepsilon, \mu, t)$ is the guiding center distribution function, $\mathbf{v}_g \equiv d\mathbf{x}_g/dt = v_\parallel \mathbf{B}/B + \mathbf{v}_D$ is the guiding center velocity, $\varepsilon \equiv mv^2/2 = mv_\parallel^2/2 + \mu B$ is the kinetic energy, $\mathcal{C}\{f\}$ is the Coulomb collision operator, and the other notation is standard.

Effects due to magnetic field line advection and diffusion of the electron guiding centers are indicated by the Fokker-Planck spatial diffusion operator ($\mathcal{D}$), which in general is [3,4] $\mathcal{D}\{f\} \equiv -\boldsymbol{\nabla\cdot} [(\langle \Delta \mathbf{x} \rangle / \Delta t) f] + \boldsymbol{\nabla\cdot} [\boldsymbol{\nabla\cdot} (\langle \Delta \mathbf{x} \Delta \mathbf{x} \rangle / 2\, \Delta t) f]$. Using $\boldsymbol{\nabla\cdot}\mathbf{A} \equiv \sum_i (1/\sqrt{g})(\partial/\partial u^i)(\sqrt{g}\mathbf{A}\boldsymbol{\cdot}\boldsymbol{\nabla} u^i)$ and the Fokker-Planck coefficients in (31), (32), when f is solely a function of a magnetic flux coordinate (i.e., ρ, ψ_* or x), the flux-suface-average of this operator becomes [neglecting $\langle \boldsymbol{\nabla}\rho \boldsymbol{\cdot} \partial \mathbf{e}_\rho/\partial\rho \rangle = \langle \boldsymbol{\nabla}\rho \boldsymbol{\cdot} \partial^2 \mathbf{x}/\partial \rho^2 \rangle \sim \mathcal{O}(\epsilon^2)$]

$$\boxed{\langle \mathcal{D}\{f(\rho)\} \rangle \simeq \frac{1}{V'} \frac{\partial}{\partial \rho} \left(- V' \bar{u}_g f + \frac{\partial}{\partial \rho} V' \bar{D}_\eta f \right).} \tag{35}$$

Next, consider Fourier expansion of the distribution function in poloidal (θ) and toroidal (ζ) angles:

$$\begin{aligned} f(\psi, \theta, \zeta) &= \sum_{m,n} f_{mn}(\psi)\, e^{im\theta - in\zeta} \equiv f_{\rm a} + f_{\rm na} \\ &= \sum_m f_{m0}\, e^{im\theta} + \sum_{m,n\neq0} f_{mn}\, e^{im\theta - in\zeta}. \end{aligned} \tag{36}$$

The $n = 0$ contributions represent the axisymmetric distribution function f_a that yields [2] the usual neoclassical transport [6,7]. The electron energy transport equation including both neoclassical and axisymmetric paleoclassical effects is obtained from the flux-surface-average of the kinetic energy moment of the axisymmetric part of (34), approximating f in $\mathcal{D}\{f\}$ by a Maxwellian $f_M(\psi)$:

$$\frac{3}{2} \frac{\partial}{\partial t} (n_e T_e) + \frac{\partial}{\partial V} \langle (\mathbf{q}_e^{\rm nc} + \frac{5}{2} T_e \boldsymbol{\Gamma}_e^{\rm nc}) \boldsymbol{\cdot} \boldsymbol{\nabla} V \rangle + \frac{\partial}{\partial V} \langle \mathbf{Q}_{\rm ea}^{\rm pc} \boldsymbol{\cdot} \boldsymbol{\nabla} V \rangle = Q_e. \tag{37}$$

Here, the electron entropy-producing processes are: neoclassical conductive ($\mathbf{q}_e^{\rm nc}$) and convective $[(5/2)T_e\boldsymbol{\Gamma}_e^{\rm nc}]$ heat fluxes, axisymmetric paleoclassical heat flux [$\mathbf{Q}_{\rm ea}^{\rm pc}$ — see (51) below] which is induced by $\mathcal{D}\{f_M\}$, and heating (Q_e) due to collisional effects (joule heating, electron viscosity, and collisions with ions).

Near a $q_* = m/n$ rational surface the nonaxisymmetric distribution function $f_{\rm na}$ can be put into a form [2] that isolates its poloidal (θ) and helical [$\alpha \equiv \zeta - q_*\theta = \zeta - (m/n)\,\theta$] angle dependences: $f_{\rm na}(\psi,\theta,\alpha) = \sum_{n\neq 0} e^{-in\alpha} \sum_{\tilde{m}} f_{m+\tilde{m},n}(\psi)\, e^{i\tilde{m}\theta}$. Further, since near a rational surface the magnetic field can be represented by its helical and magnetic shear components as $\mathbf{B} = \mathbf{B}_h + \mathbf{B}_*$, the parallel-streaming differential operator in (34) becomes [10]: $\mathbf{B}\boldsymbol{\cdot}\boldsymbol{\nabla} f = (\mathbf{B}\boldsymbol{\cdot}\boldsymbol{\nabla}\theta)[\partial f/\partial\theta|_{\psi_*,\alpha} + \;(q-q_*)\,\partial f/\partial\alpha|_{\psi_*,\theta}]$. Thus, near the $q_* \equiv m/n$ rational surface $f \to f(\psi_*,\theta,\alpha,\varepsilon,\mu)$ and applying this parallel-streaming operator to $f_{\rm na}$ yields

$$\mathbf{B}\boldsymbol{\cdot}\boldsymbol{\nabla} f_{\rm na} = (\mathbf{B}\boldsymbol{\cdot}\boldsymbol{\nabla}\theta) \sum_{n\neq 0} e^{-in\alpha} \sum_{\tilde{m}} e^{i\tilde{m}\theta} i\,[\tilde{m} - n(q-q_*)]\, f_{m+\tilde{m},n}(\psi). \qquad (38)$$

Since the parallel-streaming term $(v_\parallel/B)\,\mathbf{B}\boldsymbol{\cdot}\boldsymbol{\nabla} f_{\rm na}$ is dominant in (34), it causes the Fourier coefficients $f_{m+\tilde{m},n}$ to be small unless $\tilde{m} - n(q-q_*)$ is small. Near the $q_* \equiv m/n$ rational surface $q \simeq q_* + xq'$ and this coefficient becomes $\tilde{m} - n(q-q_*) \simeq \tilde{m} - nxq'$. It will be small and lead to the largest $f_{m+\tilde{m},n}$ for $\tilde{m} = 0$ and $|nxq'| << 1$. The resulting "helically resonant" Fourier coefficient (near $q = q_*$) will be $f_*(x) \equiv f_{m,n}(\psi_*)$. Here, the argument has been changed from the poloidal (ψ) to the helical (ψ_*) flux, which is the appropriate flux (radial) label near the given rational surface. Using (7), the criterion $|nxq'| << 1$ is $|x| << \Delta$. Hence, $f_*(x)$ solutions will be highly peaked near the q_* rational surface.

Developing a useful (i.e., one-dimensional) representation for $f_{\rm na}$ near a $q_* = m/n$ rational surface for $n >> 1$ is analogous to the development of ballooning mode theory [11,12]. The basic issue is: how does one maintain periodicity of the solutions in the poloidal (θ) and helical (α) angles as one moves radially away (i.e., to $x \neq 0$ in a sheared magnetic field structure) from a rational surface composed of helically symmetric field lines. For "flute-like" responses extending long distances ($|\ell| >> \pi\bar{R}q$) along large n helical field lines, one assumes q is locally a linear function of x (i.e., $q \simeq q_* + xq'$) and employs the procedure Lee and Van Dam [12] used to develop a ballooning representation, to obtain [2]

$$f_{\rm na} \simeq \sum_{n\neq 0} e^{-in\alpha} \sum_{p=-\infty}^{\infty} \hat{f}_*(\theta + 2\pi p)\, e^{inxq'(\theta+2\pi p)}. \qquad (39)$$

Here, $\hat{f}_*(\theta+2\pi p)$ is [12] the Fourier transform of $f_*(x)$. Note that this form of $f_{\rm na}$ is a periodic function of both poloidal (θ) and helical (α) angles. In the limit of large n the discrete sum over p can be converted [2] into a continuous integral over $\ell \simeq (\theta+2\pi p)\bar{R}q_*$ which represents extension of the poloidal angle θ into a field line variable ℓ along $\mathbf{B}$:

$$\boxed{f_{\rm na}(x,\alpha) \simeq \sum_{n\neq 0} e^{-in\alpha} f_*, \quad f_*(x) \simeq \int_{-\ell_*}^{\ell_*} \frac{d\ell}{2\pi\bar{R}q_*}\, e^{i\,k_\parallel(x)\,\ell}\, \hat{f}_*(\ell).} \qquad (40)$$

For the sheared ($q' \neq 0$) magnetic field, $k_\parallel(x) \equiv nxq'/\bar{R}q_*$.

The ℓ integration limits in (40) are the half length of a helical field line: $\ell_* = \pi\bar{R}q_*n$ from (2). These limits also imply the sum over p in (39) only extends from $-n/2$ to $n/2$ — to represent summing over n poloidal transits of the field line. Since $\hat{f}_*(\ell)$ is usually nearly constant for $|\ell| \leq \ell*$ (see discussion below (46) and [2]), (40) yields a factor $\sim \ell_*/\pi\bar{R}q_* = n >> 1$, which produces the multiplier M [see (48), (52)] in the paleoclassical electron heat diffusivity — physically because contributions of n poloidal passes of the rational helical field line are summed to obtain the net response for one poloidal period of the plasma. In the "ballooning representation" the parallel distance ℓ is proportional to the Fourier transform variable $k_x(\ell)$ for the x (radial) variation of $f_*(x)$. Also, note that $k_\parallel(x)\,\ell = k_x(\ell)\,x$, where $k_x(\ell) \equiv nq'(\ell/\bar{R}q_*) = nq'(\theta+2\pi p)$, which is the usual [11,12] $k_x = k_\theta s\,\theta$ with $k_\theta \equiv nq/\rho$ and $s \equiv \rho\,q'/q$.

Satisfying the criterion $k_x^2(\ell)\,\bar{\delta}_e^2 < 1$ (or $|x|^2 > \bar{\delta}_e^2$) for diffusing field lines [see discussion after (26) and [2]) requires $|\ell| < \ell_\delta \equiv \bar{R}q_*/(n\bar{\delta}_e q')$. Requiring ℓ_δ to be longer than the helical field line length $\ell_* \equiv \pi\bar{R}q_*n$ in (2) yields a maximum n and length of field lines that are diffusing over their entire length:

$$\boxed{n_{\max} \equiv 1/(\pi\bar{\delta}_e q')^{1/2}, \quad \text{maximum } n,} \tag{41}$$

$$\boxed{\ell_{\max} \equiv \pi\bar{R}q_* n_{\max}, \quad \text{maximum diffusing length.}} \tag{42}$$

When q' is quite small [e.g., near a minimum in q at the magnetic axis ($\rho = 0$) or off-axis ($\rho = \rho_{\min}$) for negative central magnetic shear cases], $n_{\max}$ is bounded by [2]

$$\max\{n_{\max}\} = 1/(\pi^2\bar{\delta}_e^2 q'')^{1/3}, \quad \text{near } q' = 0. \tag{43}$$

Solutions of the nonaxisymmetric MDKE in (34) are sought [2] using an ordering scheme in which the transit frequency $\omega_t \sim v_\parallel(\mathbf{B\cdot\nabla}\theta)/B \sim v_{Te}/R_0q$ is larger than all other frequencies. To lowest order $\partial f_{*0}/\partial\theta|_{\psi_*,\alpha} = 0$; hence f_{*0} must be independent of the poloidal angle θ. The next order kinetic equation includes parallel streaming along ψ_* surfaces and collisions. Bounce-averaging it annihilates a $\partial f_{*1}/\partial\theta$ term to yield $\omega_t\,(q-q_*)\,\partial f_{*0}/\partial\alpha|_{\psi_*} = \langle\mathcal{C}\{f_{*0}\}\rangle_\theta$, which in the ballooning representation becomes $\overline{v_\parallel}\,\partial\hat{f}_{*0}/\partial\ell = \langle\mathcal{C}\{\hat{f}_{*0}\rangle_\theta$. Its solution [2] (for $q - q_* \neq 0$, $\lambda_e > \ell_*$) is a Maxwellian constant along ψ_* surfaces (i.e., on closed field lines with the pitch of rational field lines):

$$\hat{f}_{*0} = f_M = n_e(\rho,t)\left(\frac{m_e}{2\pi T_e(\rho,t)}\right)^{3/2} e^{-\mathcal{E}/T_e(\rho,t)}. \tag{44}$$

When $\lambda_e < \ell_*$, finite parallel electron heat conduction [13,14] limits the electron temperature equilibration to the region $|\ell| \lesssim \lambda_e$. Thus, the $\hat{f}_{*0}$ in (44) is applicable [2] for $|\ell| \leq \ell_{f_{M*}} \equiv \min\{\ell_{\max},\,\lambda_e,\,\ell_{n^\circ}\}$.

The nonaxisymmetric paleoclassical radial electron heat transport induced by diffusion of the electron guiding centers near $q_* = m/n$ is obtained by taking the flux-surface-average of the helical Fourier average of the kinetic energy moment of the $\mathcal{D}\{\hat{f}_{*0}\}$ term from the right of (34) and using the $f_{\rm na}$ representation in (40) with $\hat{f}_{*0}$ from (44):

$$-\langle\mathbf{\nabla\cdot}\,\mathbf{Q}_{e*}^{\rm pc}\rangle \equiv \left\langle\int_{-\pi}^{\pi}\frac{d\alpha}{2\pi}\,e^{in\alpha}\!\int d^3v\,\frac{m_e v^2}{2}\frac{\partial f_{\rm na}}{\partial t}\right\rangle$$

$$\simeq \int_{-L}^{L} \frac{d\ell}{2\pi \bar{R} q_*} \, e^{ik_\|(x)\ell} \int d^3v \, \frac{m_e v^2}{2} \langle \mathcal{D}\{\hat{f}_{*0}\}\rangle. \tag{45}$$

The Fokker-Planck coefficients in (31) are only applicable for $x^2 > \bar{\delta}_e^2$. Thus, the maximum half length of helical field lines is the $\ell_{\max}$ in (42). Hence, the limits of the ℓ integration in (45) are given by $\pm L$, in which L is the minimum of the lengths over which $\hat{f}_{0*}$ is Maxwellian ($\ell_{f_{M*}}$) and field lines are diffusing ($\ell_{\max}$):

$$\boxed{L \equiv \min\{\,\ell_{\max},\,\lambda_e,\,\ell_{n^\circ}\}, \quad \text{equilibration length.}} \tag{46}$$

Since near a rational surface $|k_\|(x)L| << 1$, one can set $e^{ik_\|\ell} \simeq 1$ [for $|x| << \bar{R}q_*/nq'L \leq 1/(\pi n^2 q') = \Delta/\pi n$], perform the ℓ integration in (45), and obtain for the total paleoclassical electron heat transport near a q_* flux surface

$$\frac{\partial}{\partial V}\langle \mathbf{Q}_{e*}^{\rm pc} \cdot \boldsymbol{\nabla} V\rangle = \frac{M}{V'}\frac{\partial}{\partial \rho}\left[V'\bar{u}_g \, \frac{3}{2} n_e T_e \, - \, \frac{\partial}{\partial \rho}\left(V'\bar{D}_\eta \, \frac{3}{2} n_e T_e\right)\right], \tag{47}$$

$$\boxed{M \equiv \frac{L}{\pi \bar{R}\, q}, \quad \text{helical multiplier.}} \tag{48}$$

Considering radial profile effects [2], the net helical paleoclassical electron heat transport (after summing over all possible rational surfaces) varies slowly with radius.

In the vicinity of low order rational surfaces where $q^\circ = m^\circ/n^\circ$ with $n^\circ = 1, 2$, the factor M also varies little with radius. However, it is smaller in magnitude there because L is smaller there. The factor M remains small as q changes away from the rational surface until a distance of order $\bar{\delta}_e$ from the closest $n_{\max}$ rational surface is reached [2]. This distance can be estimated from (8), neglecting $\bar{\delta}_e << 1$ compared to $\bar{\delta}_e^{1/2}$:

$$\delta x^\circ \equiv \delta x(n^\circ) = \frac{1}{n^\circ}\left(\frac{\pi \bar{\delta}_e}{|q'|}\right)^{1/2}, \tag{49}$$

or, around a minimum in q about q°, from (9),

$$\delta x^\circ_{\min} \equiv \delta x^\circ_{\min}(n^\circ) = \left(\frac{2}{n^\circ}\right)^{2/3}\left(\frac{\pi \bar{\delta}_e}{q''}\right)^{1/3}. \tag{50}$$

5. Paleoclassical radial electron heat transport

Neoclassical [5-7] and other transport fluxes are usually specified in the rest frame of the toroidal flux sufaces. The paleoclassical heat flux is specified in this rest frame by removing the $\bar{u}_g$ contributions. For a stationary poloidal magnetic field $\mathbf{B}_p$, the total paleoclassical electron heat transport is the sum of axisymmetric ($M \to 1$ in (48) [2]) and nonaxisymmetric (quasi-helically-symmetric) transport from (48):

$$\boxed{\frac{\partial}{\partial V}\langle \mathbf{Q}_e^{\rm pc} \cdot \boldsymbol{\nabla} V\rangle = -\frac{M+1}{V'}\frac{\partial^2}{\partial \rho^2}\left(V'\bar{D}_\eta \, \frac{3}{2} n_e T_e\right).} \tag{51}$$

The diffusive part of this total paleoclassical electron heat transport indicates a paleoclassical electron heat diffusivity of

$$\boxed{\chi_e^{\rm pc} \equiv \frac{3}{2}(M+1)D_\eta \simeq \frac{3}{2}M\frac{\eta_\parallel^{\rm nc}}{\mu_0} = \frac{3}{2}M\bar{\nu}_e\delta_e^2.} \tag{52}$$

Comparing this $\chi_e^{\rm pc}$ with the magnetic flux diffusivity D_η in (23), one sees that T_e diffuses a factor of order M faster than ψ does — because T_e is equilibrated over the long length L of helically-symmetric field lines, compared to the poloidal periodicity length $\pi\bar{R}q$. Thus, the paleoclassical model may be able to explain the experimentally observed T_e "profile resiliency" [15], which was originally called "profile consistency" [16,17] and has often been linked to the q profile. For the "resilient" electron temperature profile implied by the paleoclassical model, see the discussion after (58) below. Also, the paleoclassical electron heat transport will not come to its steady-state value until the poloidal magnetic field comes into equilibium (on the resistive, "skin" diffusion time scale $\tau_\eta \sim a^2/6D_\eta$).

There are two collisionality regimes of paleoclassical electron heat diffusion. For most toroidal plasmas the electron collision length λ_e is longer than $\ell_{\rm max}$; then, $M = n_{\rm max} >> 1$ yields

$$\chi_e^{\rm pc} \simeq \frac{3}{2}\left(\frac{1}{\pi\bar{\delta}_e|q'|}\right)^{1/2}\frac{\eta_\parallel^{\rm nc}}{\mu_0}, \quad \lambda_e > \ell_{\rm max} \equiv \pi\bar{R}q\, n_{\rm max}, \tag{53}$$

which will referred to as the "collisionless" paleoclassical regime. As an example [1] of the magnitude of this paleoclassical electron heat diffusivity, for a typical ohmically-heated TFTR plasma [17] with $T_e \simeq 1.2$ keV, $n_e \simeq 3\times 10^{19}$ m^{-3}, $Z_{\rm eff} \simeq 2$, $R_0 \simeq 2.55$ m, $q \simeq 1.6$, and $a/q' \simeq 0.4$ m at $r/a \simeq 0.4/0.8 = 0.5$, one obtains $\eta_0/\mu_0 \simeq 0.067$ m^2/s, $\eta_\parallel^{\rm nc}/\eta_0 \simeq 2.2$ (neglecting ν_{*e} effects, which would make the results a factor of 0.6 smaller), $\delta_e \simeq 10^{-3}$ m, $n_{\rm max} \simeq 11$, and $\lambda_e \simeq 300$ m $> \pi R_0 q\, n_{\rm max} \simeq 140$ m, so that $L \simeq \pi R_0 q\, n_{\rm max}$, $M = n_{\rm max} \simeq 11$, and the estimated $\chi_e^{\rm pc}$ is 2.5 m^2/s $\sim \chi_e^{\rm exp}$. The radial variation of this electron heat diffusivity is due primarily to the T_e dependence of the resistivity since in the collisionless paleoclassical regime $\chi_e^{\rm pc} \propto n_e^{1/4}/(q'T_e^3)^{1/2} \propto T_e(r)^{-3/2}$ — but $\nu_{*e} \propto n_e q/\epsilon^{3/2}T_e^2$ effects in $\eta_\parallel^{\rm nc}$ can be significant for $0.1 \lesssim \nu_{*e} \lesssim 3$. Thus, $\chi_e^{\rm pc}$ usually increases with increasing r, in qualitative agreement with data from many tokamaks.

In high density, more collisional plasmas where $L = \lambda_e$, $M = \lambda_e/(\pi\bar{R}q) >> 1$ yields

$$\chi_e^{\rm pc} \simeq \frac{3}{2}\frac{\eta_\parallel^{\rm nc}}{\eta_0}\frac{v_{Te}}{\pi\bar{R}q}\frac{c^2}{\omega_p^2}, \quad \pi\bar{R}q < \lambda_e < \pi\bar{R}q\, n_{\rm max}, \tag{54}$$

which will be referred to as the "collisional" paleoclassical regime. In typical high density toroidal plasmas $Z_{\rm eff} \simeq 1$ and $\nu_{*e} > 1$; for such plasmas $(3/2)(\eta_\parallel^{\rm nc}/\eta_0) \simeq (1.5)(0.51)$. Thus, the collisional $\chi_e^{\rm pc}$ implies an overall electron energy confinement time $\tau_{Ee} \sim a^2/4\chi_e^{\rm pc} \simeq 0.27\,(n_e/10^{20}{\rm m}^{-3})\,a^2R_0q\,(T_e/500{\rm eV})^{-1/2}$ s, which approximately reproduces (in both magnitude and scaling for the highest performance pellet-fueled Alcator C plasmas [18] that had $a = 0.165$ m and $R_0 = 0.64$ m) the "neo-Alcator scaling" deduced empirically primarily from ohmically-heated tokamak plasma data in the 1970s and early 1980s [19]: $\tau_E^A \sim 0.07\, n_e a R_0^2 q_a$.

Because both ohmic heating and paleoclassical transport are proportional to the neoclassical parallel resistivity, in ohmically heated tokamak plasmas the electron power balance $\eta_\parallel^{\rm nc} J^2 = -(\partial/\partial V)\langle \mathbf{Q}_e^{\rm pc}\boldsymbol{\cdot}\boldsymbol{\nabla} V\rangle$ yields a scaling $\beta_{pe} \equiv \overline{n_e T_e}/[B_p(a)^2/2\mu_0] \sim 1/M$. This is in reasonable accord with early ohmic tokamak experimental data [20] since for the more collisional plasmas of the early 1970s $\beta_p \sim 1/M \sim \pi R_0 q/\lambda_e \sim 0.2$–1 (or $\propto n_e/I$ [20] for T_e approximately constant). For present-day, higher T_e, more collisionless tokamak plasmas that are ohmically-heated, $\beta_p \sim 1/n_{\rm max} \sim 0.1$.

In the closed field line region near but inside the magnetic separatrix region of divertor plasmas where q and q' become very large, one can have $\lambda_e \lesssim \pi \bar{R} q$. In this near-separatrix (subscript s) region, the paleoclassical electron heat diffusivity is [2]

$$\chi_{es}^{\rm pc} \simeq \frac{3}{2}\frac{\eta_\parallel^{\rm Sp}}{\mu_0}\left(1 + \frac{\eta_\parallel^{\rm nc}}{\eta_\parallel^{\rm Sp}}\frac{\lambda_e}{\pi \bar{R} q}\right), \quad \pi R < \lambda_e < \pi \bar{R} q \ \max\{1, n_{\rm max}\}. \tag{55}$$

For $\lambda_e/\pi\bar{R}q > (\eta_\parallel^{\rm Sp}/\eta_\parallel^{\rm nc}) \sim 1$, this yields the collisional $\chi_e^{\rm pc}$ in (54). In the opposite limit one obtains a smaller $\chi_{es}^{\rm pc} \simeq (3/2)(\eta_\parallel^{\rm Sp}/\mu_0) \simeq Z_{\rm eff}\,[100/T_e({\rm eV})]^{3/2}$ m^2/s. There are some experimental indications in DIII-D L-mode [21] and H-mode [22] plasmas that within about 2 cm of the separatrix $|\boldsymbol{\nabla} T_e|$ is significantly larger, which implies $\chi_e^{\rm exp}$ is reduced there. The paleoclassical model predicts that the maximum $|\boldsymbol{\nabla} T_e|$ should occur where the two terms in (55) become comparable: $q(\rho^s) \sim (\lambda_e/\pi\bar{R})\,(\eta_\parallel^{\rm nc}/\eta_\parallel^{\rm Sp}) \sim 5$–10. The indicated $\rho^s \sim 0.95$–0.98 is in reasonable agreement with where the maximum $|\boldsymbol{\nabla} T_e|$ occurs in the experiments [21,22].

The paleoclassical model applies to all types of axisymmetric current-carrying toroidal plasmas in regions where $\epsilon^2, B_p^2/B_t^2 << 1$. For $R_0 \simeq 1$ m STs with $T_e \sim 1$ keV and $n_e \sim 3\times 10^{19}$ m^{-3}, the prediction at $r/a \sim 0.5$ is $\chi_e^{\rm pc} \sim 5$–10 m^2/s, which is in reasonable agreement with experimental results [23,24]. The $\chi_e^{\rm pc}$ is large because for STs $\eta_\parallel^{\rm nc}/\eta_0 \gtrsim 3$ is large and $q' << 1$ is small in the plasma confinement region ($r/a \sim 0.5$). For quiescent RFP plasmas in the Madison Symmetric Torus (MST) Pulsed Poloidal Current Drive (PPCD) experiments [25,26], at $r/a \sim 0.3$–0.5 one obtains $\chi_e^{\rm pc} \sim 5$–10 m^2/s (large because $q < 0.2$ and $|q'| \lesssim 0.2$ are small), which is close to the effective χ_e's inferred from global ($\bar{\chi}_e^{\rm exp} \equiv a^2/4\tau_E \sim 7.5$ m^2/s [25]) and local ($\chi_e^{\rm exp} \sim$ 10–30 m^2/s [26]) measurements. In quasi-symmetric stellarator plasmas there would be no paleoclassical transport if there is no flux-surface-average parallel current $\langle \mathbf{J}\boldsymbol{\cdot}\mathbf{B}\rangle$; however, net flux-surface-average parallel currents in quasi-symmetric stellarators would apparently induce a $\chi_e^{\rm stell} \sim [\iota_J/(\iota_V + \iota_J)]\chi_e^{\rm pc}$ because while the field lines and rotational transform due to the parallel current (ι_J) would diffuse radially, those due to the vacuum fields (ι_V) would not — as can be deduced from (29) and (30) for a $\delta\psi$ composed of diffusing and spatially constant parts.

As indicated by (46), (48), and (52), the predicted $\chi_e^{\rm pc}$ is much smaller for the "short" helical field lines [see (2) and discussion thereafter] in the vicinity of low order rational surfaces with $q^\circ = m^\circ/n^\circ$: $\chi_e^{\rm pc} \sim (3/2)(n^\circ + 1)\,\eta_\parallel^{\rm nc}/\mu_0$. The estimated width of the low $\chi_e^{\rm pc}$ "electron Internal Transport Barriers" (eITBs) is $2\,\delta x^\circ$ for $q' \neq 0$, or, if q is near a minimum at the rational surface, $2\,\delta x^\circ_{\rm min}$; the distances δx° and $\delta x^\circ_{\rm min}$

are defined in (49) and (50), respectively. These barrier widths can be compared to some key tokamak results. First, as experiments in RTP [27] slowly moved highly localized electron cyclotron heating (ECH) radially outward, a "stair-step" reduction in the central T_e was observed as the ECH passed low order rational surfaces. From these provocative results it was inferred [27,28] that transport barriers existed with up to a factor of 10 reduction in χ_e over relative (to a) barrier widths of order 0.04 (0.1 for $q = 1/1$). For RTP parameters $2\,\delta x^\circ \sim 0.06$–$0.12$ (0.17 for $q = 1/1$), a bit wider but in reasonable agreement with the experimental results. Next, jumps in T_e (over radial widths ~ 0.2) have been observed in evolving DIII-D L-mode plasmas [29] as an off-axis minimum in $q(\rho, t)$ passes through low order rational surfaces. For the DIII-D parameters, $2\,\delta x^\circ_{\rm min}$ gives a similar estimate (~ 0.3) for the transport barrier width. Finally, in pioneering JT-60U experiments [30] a strong eITB created a large, localized $|\boldsymbol{\nabla} T_e|$; presuming q was near a minimum [31] at $q_{\rm min} = 3$, the width predicted by paleoclassical theory is 0.14, which is close to the experimentally inferred barrier width of about 0.11. Also, the $\chi_e^{\rm pc}$ is in approximate agreement (within experimental error bars) with $\chi_e^{\rm exp}$ outside, within, and inside the eITB.

Note that the implied paleoclassical electron heat flux in (51) is not in a normal (diffusive) Fourier heat flux law form (i.e., $\mathbf{q}_e = -\kappa_e \boldsymbol{\nabla} T_e \equiv -n_e \chi_e \boldsymbol{\nabla} T_e$). Rather, it can be written approximately (for $\partial M/\partial\rho \simeq 0$) as:

$$\langle \mathbf{Q}_e^{\rm pc}\boldsymbol{\cdot}\boldsymbol{\nabla} V\rangle = -\,V' n_e \bar{\chi}_e^{\rm pc} \frac{\partial T_e}{\partial \rho} - \langle \mathbf{q}_e^{\rm pi}\boldsymbol{\cdot}\boldsymbol{\nabla} V\rangle, \tag{56}$$

$$\langle \mathbf{q}_e^{\rm pi}\boldsymbol{\cdot}\boldsymbol{\nabla} V\rangle \equiv T_e \frac{\partial}{\partial \rho}\left(V' n_e \bar{\chi}_e^{\rm pc}\right), \quad \text{heat pinch}, \tag{57}$$

in which $\bar{\chi}_e^{\rm pc} \equiv \chi_e^{\rm pc}/\bar{a}^2$. The electron "heat pinch" flux $\langle \mathbf{q}_e^{\rm pi}\boldsymbol{\cdot}\boldsymbol{\nabla} V\rangle$ is usually positive (inward) and increases slightly with ρ, in qualitative agreement with experimental inferences from JET [15] and recent DIII-D experiments [32]. Alternatively, in qualitative agreement with experimental data from many tokamaks [33], $\langle \mathbf{Q}_e^{\rm pc}\boldsymbol{\cdot}\boldsymbol{\nabla} V\rangle$ can be written in the form of a heat flux proportional to the degree to which the electron temperature gradient exceeds a critical magnitude of

$$\left.\frac{1}{L_{Te}}\right|_{\rm crit} \equiv -\langle \hat{\mathbf{e}}_\psi \boldsymbol{\cdot}\boldsymbol{\nabla} \ln T_e\rangle \simeq \frac{\partial}{\partial r} \ln\left(V' n_e \bar{D}_\eta\right). \tag{58}$$

In tokamak plasmas with ohmic and modest auxiliary electron heating, $\boldsymbol{\nabla} \ln T_e$ in the "confinement region" ($0.3 \lesssim \rho \lesssim 0.8$) is found to be nearly constant [34], and is usually close to the inverse of a critical gradient scale length. The paleoclassical model would be in agreement with such experimental results if the critical radial gradient of $\ln T_e$ in (58) is approximately constant over the confinement region. Presuming this to be the case, the paleoclassical critical electron temperature gradient would imply a "resilient" electron temperature profile. However, since $\bar{D}_\eta$ is a complicated function of T_e and other profile-dependent parameters [mainly through the dependence of $\eta_\parallel^{\rm nc}$ on $\nu_{*e} \propto n_e q/r^{3/2} T_e^2$ — see (14) and (17)], there is apparently no simple formula for the paleoclassical-predicted resilient T_e profile.

For paleoclassical electron heat transport the effective or "power balance" χ_e ($\chi_e^{\rm pb}$), which is defined as the net electron heat flux divided by $-n_e \boldsymbol{\nabla} T_e$ (based on an assumed Fourier heat flux law), is

$$\chi_e^{\rm pb} \equiv \frac{\langle \mathbf{Q}_e^{\rm pc} \boldsymbol{\cdot} \boldsymbol{\nabla} V \rangle}{-V' n_e \partial T_e/\partial\rho} \simeq \chi_e^{\rm pc} \left(1 - \frac{\partial \ln(V' n_e \bar{D}_\eta)/\partial\rho}{-\partial \ln T_e/\partial\rho} \right). \tag{59}$$

In the usual situation where T_e decreases as the minor radius variable ρ increases but $V' n_e \bar{D}_\eta$ increases with ρ, one obtains $\chi_e^{\rm pb} < \chi_e^{\rm pc}$. This behavior is in qualitative agreement with experimental data from JET on heat pinch effects [15].

6. Paleoclassical model context, applicability, and tests

Paleoclassical electron heat transport is based on the primitive Coulomb collision processes of parallel electron heat conduction and plasma resistivity leading to magnetic field diffusion in low collisionality, current-carrying toroidal plasmas. Thus, it is an "irreducible, ubiquitous" transport process, just as classical and neoclassical transport [6,7] are. Hence, it sets a "base" level of radial electron heat transport in current-carrying toroidal plasmas — even those with a non-inductively-driven parallel current. [A corresponding paleoclassical model for particle transport has not yet been developed — because it requires development of ion paleoclassical transport, including effects due to a finite ion gyroradius (since $\varrho_i \sim \delta_e$ for many plasmas of interest), ion particle drifts, parallel density equilibration, and determination of the ambipolar potential structure.]

The origin of transport in the paleoclassical model is fundamentally different from that in transport models based on "magnetic flutter" [35] inducing (via island overlap or dissipative processes) magnetic field stochasticity and consequent radial electron heat transport with the well-known Rechester-Rosenbluth formula [36]. In particular, the diffusion of poloidal magnetic flux (field lines) caused by $\eta \langle \mathbf{J} \boldsymbol{\cdot} \mathbf{B} \rangle \neq 0$ does not induce any magnetic flutter (i.e., $\tilde{\mathbf{B}} \boldsymbol{\cdot} \boldsymbol{\nabla} \psi \neq 0$) — because the diffusing field lines always lie on flux surfaces and the solenoidal condition $\boldsymbol{\nabla} \boldsymbol{\cdot} \mathbf{B} = 0$ is satisfied automatically at all times by the flux function representations of $\mathbf{B}$. Thus, there is no magnetic-flutter-induced transport in the paleoclassical model. Also, the Fokker-Planck diffusion coefficient is different. Instead of $\langle (\Delta x)^2 \rangle / 2\Delta t = D_\eta = \eta_\parallel^{\rm nc}/\mu_0$ as given in (31), for a stochastic magnetic field the appropriate Fokker-Planck diffusion coefficient is $\langle (\Delta x)^2 \rangle / 2\Delta t = |v_\parallel| D_m$ in which the stochastic magnetic field diffusion coefficient is [36] $D_m \equiv \langle (\Delta x)^2 \rangle / 2\Delta \ell = \sum_k (\tilde{B}_{xk}/B)^2 \delta(k_\parallel)$. The difference in the diffusion coefficient is important for runaway electrons with speeds $v_R >> v_{Te}$ because the implied runaway diffusion coefficient for the Rechester-Rosenbluth model is $D_R^{RR} \sim v_R D_m$, i.e., proportional to the runaway speed v_R. In contrast, for the paleoclassical model $D_R^{\rm pc} \simeq (2/3)\, \chi_e^{\rm pc}$ is independent of v_R. Experimental tests in tokamak [37,38] and RFP [26] plasmas have indicated runaway diffusion coefficients much smaller than D_R^{RR}, and perhaps of order $\chi_e^{\rm exp}$, in qualitative agreement with the paleoclassical prediction $D_R^{\rm pc}$.

Plasma turbulence produced by microinstabilities induces additional transport. In high electron temperature toroidal plasmas, since $D_\eta \propto \eta \propto 1/T_e^{3/2}$, the paleoclassical electron heat diffusion coefficient $\chi_e^{\rm pc}$ usually decreases with increasing T_e roughly as $a^{1/2}/T_e^{3/2}$ (neglecting ν_{*e} effects which weaken the inverse T_e scaling). For $n_{\rm max} \lesssim 10$ and $\eta_\parallel^{\rm nc}/\eta_0 \lesssim 2$, $\chi_e^{\rm pc}$ becomes less than 1 m^2/s for $T_e \gtrsim 2$ keV. In contrast, since microturbulence-induced diffusion tends to be a multiple of the gyro-Bohm coefficient $\chi^{\rm gB} \equiv (\varrho_s/a)(T_e/eB) \propto T_e^{3/2}/aB^2$, it tends to increase with T_e. Thus, if plasma microinstabilities induce $\chi_e^{\mu{\rm turb}} \gtrsim 1$ m^2/s for $T_e \gtrsim 2$ keV, they would become the dominant transport mechanisms as electron temperatures increase above a few keV. However, when microinstabilities are stabilized or the transport they induce is strongly reduced (e.g., by sheared $\mathbf{E}\times\mathbf{B}$ flows or in transport barriers), the paleoclassical electron heat transport would still set the base transport level, even at high T_e. Also, since $\chi_e^{\rm pc}/\chi^{\rm gB} \propto a^{3/2}B^2/T_e^3$, paleoclassical radial electron heat transport is likely to be dominant toward and in the cooler edge of all toroidal plasmas — perhaps wherever the electron temperature T_e is less than about 1 keV$\times B({\rm T})^{2/3}a({\rm m})^{1/2}$.

Because the physical processes underlying paleoclassical and microturbulence-induced electron heat transport are so different, they may be mostly independent of each other; hence, the transport they induce might just be additive. Experimentally, the parallel Ohm's law seems to be well represented by the neoclassical predictions for both axisymmetric poloidal [39] and helically-symmetric [40] magnetic field evolution. Theoretically, paleoclassical transport processes would apparently not be directly affected by microturbulence since: 1) microturbulence usually does not significantly affect the parallel Ohm's law [41,42] because its parallel wavenumbers and hence momentum transfer are small; and 2) the parallel correlation length for magnetic microturbulence usually exceeds the relevant paleoclassical length L.

To distinguish between various types of radial electron heat transport processes it would be useful if experimentalists could begin referencing their inferred $\chi_e^{\rm pb}$ values to the characteristic diffusivities for the various possible mechanisms for χ_e:

$$D_{\eta_0} \equiv \frac{\eta_0}{\mu_0} \simeq 0.044\,\frac{Z_{\rm eff}}{[T_e({\rm keV})]^{3/2}}\,\frac{{\rm m}^2}{\rm s}, \tag{60}$$

$$\chi_i^{\rm gB} \equiv \frac{\varrho_S}{a}\frac{T_e}{eB} \simeq 3.2\,\frac{[T_e({\rm keV})]^{3/2}A_i^{1/2}}{a({\rm m})\,[B({\rm T})]^2}\,\frac{{\rm m}^2}{\rm s}, \tag{61}$$

$$\chi_e^{\rm gB} \equiv \frac{\varrho_e}{a}\frac{T_e}{eB} \simeq 0.075\,\frac{[T_e({\rm keV})]^{3/2}}{a({\rm m})\,[B({\rm T})]^2}\,\frac{{\rm m}^2}{\rm s}, \tag{62}$$

in which the "ion sound" and electron gyroradii are defined by $\varrho_S \equiv (T_e/m_i)^{1/2}/\omega_{ci}$ and $\varrho_e \equiv (T_e/m_e)^{1/2}/\omega_{ce}$, respectively. The paleoclassical $\chi_e^{\rm pc}$ is predicted to be a multiple (typically $\sim$ 10–30 >> 1) of the magnetic field diffusivity D_η and hence should be referenced to it, or for simplicity to the reference diffusivity D_{η_0}. Ion-temperature-gradient (ITG) driven and other drift-wave-type (e.g., trapped electron) microturbulence typically lead to predictions for the electron $\chi_e^{\mu{\rm turb}}$ that are a multiple (typically $\sim$ 1–3) of the ion sound gyro-Bohm diffusivity $\chi_i^{\rm gB}$. Finally, electron-temperature-gradient (ETG) microturbulence [43] leads to χ_e predictions that are multiples [perhaps as large as 48 [44,45], but maybe only of order 3–10 [46–49] for the $a/L_{Te} = (a/R_0)(R_0/L_{Te}) =$

$0.36 \times 6.9 \simeq 2.5$, $a/L_n = (a/R_0)(R_0/L_n) = 0.36 \times 2.2 \simeq 0.8$ cases often explored in those papers] of the electron gyro-Bohm diffusivity $\chi_e^{\rm gB}$. Hence, determining how large the experimentally inferred χ_e's are relative to the characteristic diffusion coefficients $D_{\eta 0}$, $\chi_i^{\rm gB}$ and $\chi_e^{\rm gB}$ given in (60)–(62) may yield clues about the type of physical processes (paleoclassical, drift-wave, or ETG type) responsible for radial electron heat transport — particularly for low power density, low electron density, and/or $T_e > T_i$ regimes where the electron and ion heat transport channels are weakly coupled and the electron transport processes can be explored somewhat independently.

Transport scalings for magnetically confined plasmas are often sought in terms of dimensionless physical variables [50–52]. The local diffusivities are usually scaled relative to the gyro-Bohm diffusivity $(\varrho/a)(T_e/eB)$, as in (61), (62); the global energy confinement time ($\tau_E \sim a^2/4\chi$) is usually scaled relative to the gyrofrequency $\omega_c \equiv qB/m$. Thus, in terms of physically relevant dimensionless variables for magnetized toroidal plasmas such as a normalized gyroradius $\varrho_* \equiv \varrho/a$, relative pressure $\beta \equiv P/(B^2/2\mu_0)$, and neoclassical collisionality ν_{*e}, the scalings are usually of the form $\chi/\chi^{\rm gB} \sim h_{\rm gB}(\varrho_*, \beta, \nu_{*e})$ and $\omega_c \tau_E \sim f_{\rm gB}(\varrho_*, \beta, \nu_{*e})$. While these are the relevant scalings and dimensionless physical variables for transport induced by drift-wave and ETG microturbulence, they are not appropriate for paleoclassical transport.

The natural parameters against which the paleoclassical local diffusivity $\chi_e^{\rm pc}$ and global electron energy confinement time $\tau_{Ee} \sim a^2/4\chi_e^{\rm pc}$ should be scaled are the magnetic field diffusivity D_η and the resistive or skin time $\tau_\eta \sim a^2/6D_\eta$. Thus, in terms of the paleoclassical-relevant physical parameters of the normalized em skin depth $\bar{\delta}_e$ and neoclassical electron collisonality ν_{*e}, their natural scalings are:

$$\frac{\chi_e^{\rm pc}}{\eta_0/\mu_0} \equiv \frac{\chi_e^{\rm pc}}{\nu_e \delta_e^2} \sim f_{\rm pc}(\bar{\delta}_e, \nu_{*e})\, g_{\rm pc}(q', q, \epsilon), \tag{63}$$

$$\frac{\tau_{Ee}^{\rm pc}}{\tau_\eta} \sim \overline{f_{\rm pc}^{-1}}(\bar{\delta}_e, \nu_{*e})\, \overline{g_{\rm pc}^{-1}}(q', q, \epsilon), \tag{64}$$

in which $f_{\rm pc}$ and $g_{\rm pc}$ are functions of the relevant dimensionless physical and geometric variables, respectively; $\overline{f_{\rm pc}^{-1}}$ and $\overline{g_{\rm pc}^{-1}}$ are appropriate spatial averages (e.g., as in [15]) of the reciprocals of these functions. In the normally applicable collisionless paleoclassical regime (53) one has $f_{\rm pc} \sim (\bar{\delta}_e)^{-1/2}$ and $g_{\rm pc} \sim (q')^{-1/2}$ for $\nu_{*e} << 1$; a ν_{*e} dependence of $f_{\rm pc}$ and a dependence of $g_{\rm pc}$ on q and ϵ can develop both from the neoclassical parallel resistivity (14) or as one moves into the collisional paleoclassical regime (54), usually toward the cooler plasma edge.

In terms of the natural variables for paleoclassical transport, in the normal collisionless paleoclassical regime the local electron heat diffusivity and electron energy confinement time depend on only the one dimensionless physical parameter $\bar{\delta}_e$ — via the dependence of $f_{\rm pc}$ on it. Perhaps this explains why electron heat transport is so ubiquitous and does not change much between various ohmically-heated toroidal plasmas — where the plasma resistivity plays a key role in both ohmic heating and paleoclassical transport. Finally, it should be noted that, as with neoclassical transport, the paleoclassical scaling laws in (63) and (64) probably cannot be probed by step increases in auxiliary electron heating power that successively increase T_e since in such a scenario $D_{\eta 0} \sim 1/T_e^{3/2}$ decreases whereas microturbulence-induced transport, which scales as $\chi^{\rm gB} \sim T_e^{3/2}$, increases and is likely to become the limiting transport mechanism.

Since the magnetic field does not appear explicitly in any of the parameters in (63) and (64), one might wonder how they could possibly apply to "magnetically confined" plasmas? However, the magnetic field is implicitly involved since D_η represents the radial diffusion of the magnetic field and q represents the ratio of the poloidal to toroidal magnetic field. Thus, paleoclassical transport processes do involve the magnetic field, albeit implicitly, and hence are applicable to all current-carrying magnetically-confined toroidal plasmas.

The present paleoclassical analysis is limited [2] to near-equilibrium, "quasi-stationary" situations where ψ changes so little in space and time that $|\partial\dot{\psi}/\partial\rho| << |(q'/q)\dot{\psi}|$ and $|\dot{\psi}| << \bar{D}_\eta\,|q'\psi'/q_*|$, in which $\dot{\psi} \equiv d\psi/dt$ — so field lines advect and diffuse radially more than temporal changes in the poloidal magnetic flux cause rational surfaces and field lines to move radially. More physically, these criteria imply that at all radii in the plasma the spatial and temporal changes in the inductive toroidal electric field $\langle \delta\mathbf{E}\boldsymbol{\cdot}\mathbf{e}_\zeta\rangle \equiv \langle \delta\mathbf{E}\boldsymbol{\cdot} R^2\boldsymbol{\nabla}\zeta\rangle \simeq \langle \delta\mathbf{E}\boldsymbol{\cdot}\mathbf{B}\rangle/\langle \mathbf{B}\boldsymbol{\cdot}\boldsymbol{\nabla}\zeta\rangle$ must be relatively homogeneous spatially and small compared to the equilibrium inductive electric field $\langle \mathbf{E}_0\boldsymbol{\cdot}\mathbf{B}\rangle \equiv \eta_\parallel^{\rm nc}\langle \mathbf{J}_0\boldsymbol{\cdot}\mathbf{B}\rangle$ needed to drive the flux-surface-averaged parallel current $\mathbf{J}_0$ in equilibrium:

$$\partial\langle \delta\mathbf{E}\boldsymbol{\cdot}\mathbf{B}\rangle/\partial\rho << (q'/q)\langle \delta\mathbf{E}\boldsymbol{\cdot}\mathbf{B}\rangle, \quad \langle \delta\mathbf{E}\boldsymbol{\cdot}\mathbf{B}\rangle << \eta_\parallel^{\rm nc}\langle \mathbf{J}_0\boldsymbol{\cdot}\mathbf{B}\rangle. \tag{65}$$

Many types of transient transport experiments apparently violate these criteria; hence, they cannot be analyzed with the present quasi-stationary paleoclassical model.

With the preceding limitations in mind, various types of local tests of the paleoclassical model are possible and suggested for quasi-stationary plasmas in which conditions (65) are satisfied:

A) Electron Heat Diffusivity: 1) Is $\chi_e^{\rm pc}$ close to the experimentally inferred $\chi_e^{\rm pb}$ in magnitude?; and 2) Do their radial profiles agree? It would be helpful if $\chi_e^{\rm pc}$ could be routinely evaluated and displayed in plots of $\chi_e^{\rm pb}$ versus ρ. (As a first step, the effects of low order rational surfaces in $\chi_e^{\rm pc}$ could be neglected for simplicity.)

B) Electron Internal Transport Barriers (eITBs): 1) Do eITBs occur at all low order rational surfaces where $q^\circ \equiv m^\circ/n^\circ$ with $n^\circ = 1, 2$?; 2) Do the widest eITBs occur around minima in q about q°? 2) Do the radial barrier widths agree with $2\delta x^\circ$ from (49) [or $2\delta x^\circ_{\rm min}$ from (50) if q° is at a minimum in q]?; 3) Is $\chi_e^{\rm pc}$ approximately constant within the eITB but then increases rapidly a few δ_e from the nearest $n_{\rm max}$ rational surface?; and 4) Are the barrier depths in agreement with the paleoclassical prediction of $\chi_e^{\rm pc}$(in eITB)/$\chi_e^{\rm pc}$(outside eITB) $\sim (n^\circ + 1)/n_{\rm max} \sim 1/5$?

C) Electron Heat Flux: 1) In cases where the paleoclassical electron heat transport is likely to be dominant ($T_e \lesssim 1$ keV$\times B^{2/3}a^{1/2}$), is the experimental heat transport data approximately represented by (56) — i.e., with a heat pinch or minimum temperature gradient form; 2) If so, does the magnitude of the electron heat pinch agree with (57) — or the minimum T_e gradient agree with (58)?; 3) Is the $1/L_{Te}|_{\rm crit}$ in (58) nearly constant over the main confinement region of $0.3 \lesssim \rho \lesssim 0.8$?; and 4) Is the electron heat transport ever less than the paleoclassical electron heat transport given in (51)?

D) Non-tokamak Experiments (in regions where $\epsilon^2, B_p^2/B_t^2 << 1$): 1) Is the large χ_e in low aspect ratio ST plasmas well repesented by $\chi_e^{\rm pc}$ and due primarily to the large $\eta_\parallel^{\rm nc}$ or small q', or a combination of them?; 2) Is the large χ_e in quiescent RFP plasmas well-represented by $\chi_e^{\rm pc}$ and due primarily to the small q and q'?; 3) Is χ_e in spheromaks well represented by $\chi_e^{\rm pc}$?; and 4) Is $\chi_e \sim [\iota_J/(\iota_V + \iota_J)]\chi_e^{\rm pc}$ in quasi-symmetric, current-carrying stellarator plasmas?

E) Fundamental Physics Tests: 1) Are both the poloidal and helical magnetic flux difffusion governed by the neoclassical parallel resistivity, as initial indications seem to suggest [39,40]; 2) Just inside a magnetic separatrix where $\lambda_e < \pi R q$ is $\chi_e \simeq (3/2) D_\eta$?; 3) Do runaway electrons diffuse radially with a diffusion coefficient $D_R \simeq (2/3)\,\chi_e^{\rm pc}$?; and 4) Most generally, is $\chi_e \sim (L/\pi \bar{R} q) D_\eta$ with $L = \min\{\ell_{\rm max}, \lambda_e, n^\circ\}$ for each of the limiting cases for L?

As indicated by the discussion in the previous section, many "back-of-the-envelope" type tests have already been carried out with mostly encouraging results. What's needed now are more precise and detailed tests by experimentalists and transport modelers. In addition, it is important to determine the parameter regimes over which paleoclassical electron heat transport is dominant versus those regimes in which microturbulence-induced anomalous transport is dominant. In making these various tests it should be kept in mind that because the paleoclassical results were obtained [2] using a large n asymptotic analysis and the characteristic lengths in L have been determined only approximately, M (and hence all M-dependent results) should be interpreted as scaling results. Thus, numerical coefficients of the order of up to a factor of say 2 (up or down) should be allowed for in the various components of L, and hence in M and $\chi_e^{\rm pc}$.

7. Summary

Equations (51)–(59) summarize the main features of the paleoclassical model [1,2] for radial electron heat transport in current-carrying toroidal plasmas that satisfy the quasi-stationarity conditions in (65). Because they were obtained [2] by a large n asymptotic analysis and the characteristic lengths in L have been determined only approximately, M (and hence all M-dependent results) should be interpreted as scaling results with order unity numerical coefficients. As indicated in Section 5, the paleoclassical electron heat transport model provides interpretations for many features of "anomalous" electron heat transport: magnitude and radial profile of electron heat diffusivity (in tokamaks, STs, and RFPs), Alcator scaling in high density plasmas, a scaling of $\beta_{pe} \sim 1/M$ for ohmically heated plasmas, widths and depths of electron transport barriers around low order rational surfaces and near a separatrix, and a natural heat pinch (or minimum temperature gradient which could lead to a "resilient" T_e profile) electron heat flux form.

Paleoclassical transport provides a ubiquitous, irreducible and hence base level of radial electron heat transport. However, as discussed in Section 6, it probably does not provide the limiting electron heat transport process with strong auxiliary heating of electrons. Microturbulence-induced transport adds to the paleoclassical levels and likely becomes dominant at high T_e [$\gtrsim 1$ keV$\times B(\mathrm{T})^{2/3} a(\mathrm{m})^{1/2}$]. The relative magnitudes and roles of paleoclassical, neoclassical and microturbulence-induced transport remain to be delineated experimentally; Section 6 provides suggestions for how they could be explored. Section 6 also suggests a number of tests of the paleoclassical electron heat transport model that range from fundamental properties to macroscopic plasma behavior.

Acknowledgments

The author is grateful to many of his colleagues for their comments and constructive suggestions in response to seminars and talks he has given on this research at University of Wisconsin-Madison, General Atomics (San Diego), Massachusetts Institute of Technology, Princeton Plasma Physics Laboratory and Oak Ridge National Laboratory, as well as at recent Sherwood, Transport Task Force, IAEA and DPP-APS meetings. He is particularly grateful to M.E. Austin, T.H. Osborne and D.G. Whyte for sharing and discussing their respective unpublished DIII-D data, to C.C. Petty and W.A. Houlberg for clarifying discussions on dimensionless variable scaling studies, and to H.E. St. John for trying out the paleoclassical model in the ONETWO code and highlighting the $q' \simeq 0$ problem that is resolved in (43). Finally, he is grateful to the U.S. Department of Energy for support of his research over the past three decades.

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