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Scaling of Ohmic Tokamak Error-field Penetration Thresholds in the Presence of Neoclassical Toroidal Viscosity

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Abstract

A model for field error penetration is developed that includes non-resonant as well as the usual resonant field error effects. The non-resonant components cause a neoclassical toroidal viscous torque that tries to keep the plasma rotating at a rate comparable to the ion diamagnetic frequency. The new theory is used to examine resonant error-field penetration threshold scaling in ohmic tokamak plasmas. Compared to previous theoretical results, we find the plasma is *less* susceptible to error-field penetration and locking, by a factor that depends on the non-resonant error-field amplitude.

1. Introduction

Efforts to understand the penetration of non-axisymmetric magnetic field perturbations—“error-fields”—into high temperature plasmas have concentrated on the role of resonant components. In this work, it is shown that non-resonant magnetic field perturbations can play a crucial role in the error-field penetration problem by producing a global neoclassical torque that damps toroidal flow toward a diamagnetic ion-type flow. In contrast, a resonant perturbation produces a localized electromagnetic braking torque at its respective resonant surface. Accounting for both these effects leads to a criterion for the error-field penetration which indicates that the critical resonant error-field amplitude increases with plasma density, a result that is in better qualitative agreement with empirical scaling [1].

2. History and Motivation

Considerable theoretical [2–5] and experimental [1, 6–10] effort has been aimed at understanding the effects of small resonant helical magnetic field errors—arising from field coil misalignments and non-axisymmetric coil feed-throughs—on plasma confinement in tokamaks. The impetus for this research has come from the experimental correlation between the emergence of locked modes and disruptions in tearing-stable low-density ohmic discharges. Error-field locked modes are induced and develop as follows [1, 6]: 1) the resonant error field is ramped up linearly or the electron density is ramped down slowly (> 100 ms), 2) when the locked mode threshold is reached, a rapid (~ 5 ms) bifurcation to a non-rotating “locked-state" is observed, and then 3) for ~ 100 ms a stationary magnetic island—driven by the error field—develops (usually on the $q = 2$ surface) and leads to either a major disruption or confinement degradation. Locked mode avoidance in low-density ohmic discharges is highly desirable—if not crucial—for reliable tokamak operation.

To date, the theoretical and experimental error-field studies have been confined to predicting the resonant (e.g., $m/n = 2/1$) critical error-field strength (as a function of plasma density, toroidal field strength, and other variables) when bifurcation occurs and after which a locked mode develops. Currently, empirical and theoretical locked mode thresholds do not agree on the scaling to larger devices. Predictive capability for locked-mode avoidance on ITER [11] is needed. The present benchmark scenario for ITER relies on an ohmic start-up with an anticipated low toroidal rotation rate (~ 0.5 kHz).

3. Conventional Error-field Theory

The standard model [3–5, 12] employed to describe error-field penetration considers the response of a large aspect ratio toroidally-rotating tearing-stable plasma to a single resonant helical magnetic perturbation. The plasma is approximated by a periodic cylinder, with nearly circular flux surfaces. Standard cylindrical coordinates (r, θ, z) and simulated toroidal coordinates (r, θ, ϕ) with $z = R_0\phi$ will be employed in this work. The resonant field component exerts an electromagnetic torque on the plasma only in the vicinity of its rational surface [3]. This torque is brought about by the nonlinear interaction of error-field-induced eddy-currents in a singular layer around the rational surface with the error-field itself and is directed against the flow, trying to brake the plasma. Theoretical predictions of the eddy current response in the layer depend on the physics model employed. The standard model assumes a flow drive plus a phenomenological diffusive perpendicular viscous torque that opposes the electromagnetic braking torque, trying to maintain the plasma flow profile. The steady-state balance between electromagnetic and viscous torques yields a transcendental equation whose roots give the modified layer velocity (in the presence of the resonant error-field) as a function of error-field strength. Above a critical error-field strength the electromagnetic torque on the resonant surface exceeds the perpendicular viscous torque on the plasma flow, and the rational surface bifurcates to a stationary, or *locked* state. This bifurcation is termed *error-field penetration*, and the critical error-field strength at which it occurs is known as the *penetration threshold*. After locking, magnetic reconnection on the resonant surface proceeds unhindered, as if there were no equilibrium plasma flow. This scenario closely mimics observations of error-field penetration occurring during the ohmic start-up phase of several tokamaks [1, 6–10].

4. Neoclassical Toroidal Viscosity

While resonant components of the magnetic field perturbation spectrum have dominated the theoretical discussion, in tokamak experiments many non-resonant components are present and they can be larger in magnitude than the resonant components. While the non-resonant components in and of themselves cannot produce locking, these components can have a profound effect on the plasma through their role in damping the toroidal flow by a neoclassical viscous torque mechanism. Recent experiments on NSTX with large applied non-resonant magnetic perturbations demonstrated qualitative and quantitative agreement [13] with theoretical predictions [14] of toroidal flow damping.

In the context of fluid theory, Neoclassical Toroidal Viscosity [NTV] can be understood as the toroidal drag force experienced by the plasma moving along distorted flux surfaces having broken toroidal symmetry. We will consider the drag induced by an error field consisting of one resonant (i.e., $m = 2, n = 1$) and many non-resonant harmonics. Assuming the error-field-induced distortion within the toroidal plasma is small enough that the flux surface remains intact on average, we may employ the theoretical formulation of Shaing [14–16]. On each flux surface, the magnetic field strength is decomposed into helical harmonics in Hamada coordinates (Θ, ζ):

$$B = B_0\left(1 + \sum_{(n',m')\neq(0,0)} [b_{n'm'}/B_0]\, e^{i(m'\Theta - n'\zeta)}\right). \tag{1}$$

The toroidal momentum dissipation arising from NTV is described through the toroidal component of the ion viscous stress tensor and leads to a toroidal flow velocity evolution equation of the form [14]

$$\partial_t \left\langle \vec{e}_\phi \cdot \vec{V} \right\rangle = -\left\langle (1/\rho_m)\vec{e}_\phi \cdot \vec{\nabla}\cdot \overleftrightarrow{\Pi} \right\rangle + \cdots\, , \tag{2}$$

where ρ_m is the mass density, $\vec{e}_\phi$ is the covariant basis vector pointing in the toroidal direction, $\overleftrightarrow{\Pi}$ is the ion viscous stress tensor, and $\langle \cdots \rangle$ denotes a flux surface average. Evaluating the NTV force in the low collisionality $(1/\nu)$ regime the NTV force yields [13, 14]

$$\left\langle (1/\rho_m)\vec{e}_\phi \cdot \vec{\nabla}\cdot \overleftrightarrow{\Pi} \right\rangle = \nu_\parallel \left(b_{\text{eff}}^{1/\nu}\right)^2 \left(V_\phi - V_*^{NC}\right), \tag{3}$$

where

$$\left(b_{\text{eff}}^{1/\nu}\right)^2 \simeq 1.74\, B_\phi\, (R_0 q)^2\, \epsilon^{3/2} \left\langle \frac{1}{B_\phi} \right\rangle \left\langle \frac{1}{R^2} \right\rangle \sum_{(n',m')\neq(0,0)} \left|\frac{n' b_{n'm'}}{B_0}\right|^2 W_{n'm'}. \tag{4}$$

Here $V_\phi \equiv \vec{e}_\phi \cdot \vec{V}$ is the toroidal flow speed, R is the major radius, R_0 is the major radius of the magnetic axis, r is the minor radial coordinate, $\epsilon = r/R_0$, $\nu_\parallel = \omega_{ti}^2/\nu_i$, $\omega_{ti} \equiv v_{ti}/(R_0 q)$ is the ion transit frequency, ν_i is the ion-ion collision frequency, and the dimensionless coefficients $W_{m'n'}$ are defined in [13]. This regime is valid provided $\omega_E < \nu_i/\epsilon < \sqrt{\epsilon}\,\omega_{ti}$, where ω_E is the $E \times B$ drift frequency. Also, for the $1/\nu$ regime [14]

$$V_*^{NC} \simeq \frac{3.5\, R_0 q}{Z_i e\, r B_0} \frac{dT_i}{dr}, \tag{5}$$

where $Z_i e$ is the charge of the ion species. Taking the large aspect ratio limit of (3), an effective NTV torque may be included in the standard cylindrical model as:

$$T_\phi^{NC} = -R_0 \nu_\parallel\, b^2(r) \left[V_\phi(r) - V_*^{NC}(r)\right], \tag{6}$$

where

$$b^2(r) \equiv \left(b_{\text{eff}}^{1/\nu}\right)^2 \propto q^2(r)\epsilon^{3/2} \sum_{(n',m')\neq(0,0)} \left|\frac{n' b_{n'm'}}{B_0}\right|^2 W_{n'm'}. \tag{7}$$

These formulae are valid in the $1/\nu$ regime [14], typically the governing regime at high temperature. While other collisionality regimes yield different forms for b_{eff} and $V_*^{NC}(r)$, the structure of the NTV force is essentially the same as given by (3) and will be covered in a subsequent work.

5. Steady-state Toroidal Flow Profile

In the following, dimensionless quantities are employed with all length-scales normalized to r_s, the resonant-surface minor radius. The major and minor radii of the plasma are R_0 and a (normalized to r_s), respectively. The magnetic field is normalized to $B_l \equiv s(r_s) B_\theta(r_s)$, where $s(r_s) = (d \ln q / d \ln r)_{r_s}$ represents the magnetic shear at the resonant surface. Here, $q(r) \simeq r B_0 / R_0 B_\theta(r)$ is the safety-factor profile. All time-scales are normalized to $\tau_l = (r_s/V_l)$, where $V_l = B_l / \sqrt{\mu_0 \rho_m(r_s)}$, and $\rho_m(r_s)$ is the mass density at the resonant surface.

In the absence of field errors, the equilibrium toroidal flow is assumed to be supported against perpendicular viscous damping with the boundary at $r = a$ by an unspecified force, F_0. Thus, equilibrium momentum balance takes the form

$$\frac{1}{r}\frac{d}{dr}\left[\mu(r) r \frac{dV_\phi^0}{dr}\right] = -F_0. \tag{8}$$

The solution satisfying $V_\phi(a) = 0$ and $V_\phi(1) = V_0$ is:

$$V_\phi^0(r) = V_0 \left[\int_1^a \frac{x dx}{\mu(x)}\right]^{-1} \int_r^a \frac{x dx}{\mu(x)}. \tag{9}$$

Here, $\mu(r)$ is the (phenomenological) ion perpendicular viscosity [normalized to $V_l r_s \rho_m(r_s)$] that represents cross-field momentum transport due to collisional effects or microturbulence. Expressed in terms of V_0, the driving force is $F_0 = 2V_0 [\int_1^a x dx/\mu(x)]^{-1}$.

In the presence of static error fields, two additional forces enter the toroidal momentum balance equation. The first—a resonant electromagnetic torque—is strongly localized around the resonant surface and can be represented by $F_{EM}\delta(r-1)/r$, where $\delta(r-1)$ is the Dirac delta function. (The coefficient F_{EM} must be resolved using boundary layer analysis on the resonant surface and will be specified in what follows.) The second force arises from NTV and is given by (6). Thus, the new toroidal momentum balance equation is given by

$$\frac{1}{r}\frac{d}{dr}\left(\hat{\mu}(r) r \frac{dV_\phi(r)}{dr}\right) - \hat{b}^2(r)\Gamma_s^2 \left[V_\phi(r) - V_*^{NC}(r)\right] = -\frac{F_{E,M}}{\mu_s}\frac{\delta(r-1)}{r} - \frac{F_0}{\mu_s}, \tag{10}$$

where $\hat{\mu} = \mu(r)/\mu_s$, $\mu_s = \mu(r_s)$, $\hat{b}(r) = b(r)/b(r_s)$, and

$$\Gamma_s = \sqrt{\nu_\| \tau_l / \mu_s}\, b(r_s). \tag{11}$$

The parameter Γ_s determines whether perpendicular (anomalous or collisional) viscosity dominates over parallel (neoclassical toroidal) viscosity [NTV] in the bulk plasma. In the limit $\Gamma_s \ll 1$, NTV is negligible and the previous drift-MHD theory is obtained [5]. In the opposite limit $\Gamma_s \gg 1$, NTV dominates over perpendicular viscosity, and a WKB-type [17] Green function can be used to find the solution of (10) above [18]:

$$V_\phi(r) \simeq \left[V - V_*^{NC}(1)\right] \frac{\exp\left(-\Gamma_s |1-r|\right)}{\sqrt{r \hat{b}(r) \sqrt{\hat{\mu}(r)}}} + V_*^{NC}(r). \tag{12}$$

Here, $V_*^{NC}(1)$ is given by (5) evaluated at the resonant surface (and normalized to V_l described above).

6. Resonant Surface Torque Balance

The error-field penetration threshold is obtained by integrating the toroidal torques across the resonant surface [4] (i.e., $\int\int\int r\,dr\,dz\,d\theta R_0 \{ \ (10) \ \}$). Inspection of (10) and (12) reveals that the neoclassical layer torque ($T_{\phi,NTV}$) and perpendicular viscous torque ($T_{\phi,VS}$) satisfy $T_{\phi,NTV} \simeq \delta\,\Gamma_s\,T_{\phi,VS}$, where $\delta \ll 1$ is the linear layer thickness. We assume

$$1 \ll \Gamma_s \ll \frac{1}{\delta}, \tag{13}$$

which guarantees NTV may be neglected within the resonant layer, but dominates perpendicular viscosity in the bulk plasma. This constraint has two consequences: 1) as in previous drift-MHD work [5], the resonant layer toroidal torque balance expression is still between (albeit modified) perpendicular viscous and electromagnetic torques [i.e., $T_{\phi,VS} + T_{\phi,EM} = 0$]; and 2) we can use the previous drift-MHD analysis [5] to evaluate the plasma response in the resonant layer.

The layer response function is given by $\Delta = \partial \ln[b_{r,nm}(r)]/\partial r|_{1-}^{1+}$. For consistency with layer results in [5], we define the Lundquist number as $S = \tau_R/\tau_H$, where $\tau_R = \mu_0 r_s^2/\eta(r_s)$ and $\tau_H = (R_0\sqrt{\mu_0\rho_m(r_s)})/[ns(r_s)B_\phi] = \tau_l/m$. Here $\eta(r_s)$ is the (dimensional) parallel neoclassical resistivity at the resonant surface. Similar to [5] we define dimensionless frequencies $Q = S^{1/3}\omega\tau_H$, $Q_*^{NC} \sim [R_0 m/(r_s n)]S^{1/3}\omega_{*,i}\,\tau_H$, and a scaled plasma response parameter $\hat{\Delta} = S^{-1/3}\Delta$. Here $\omega = mV_{\theta,0}/r_s - nV/R_0$ is the (dimensional) resonant surface frequency in the presence of resonant and non-resonant error-fields, and $\omega_{*,i}$ is the (dimensional) ion diamagnetic flow frequency at the resonant surface. The new steady-state torque balance equation for the resonant layer ($T_{\phi,EM} + T_{\phi VS} = 0$) when $\Gamma_s \gg 1$ is [18]

$$\left|\frac{b_{r,nm}^{\text{vac}}}{B_\phi}\right|^2 \frac{\text{Im}\{\hat{\Delta}(Q)\}}{|\alpha + \hat{\Delta}(Q)|^2} = \frac{P}{\kappa S}(Q_*^{NC} - Q), \tag{14}$$

where $\kappa \equiv 1/([s(r_s)]^2\Gamma_s)$, and $b_{r,nm}^{\text{vac}}$ is the *vacuum* radial magnetic perturbation associated with the resonant error-field component (at the resonant surface). As in [5] $\alpha \equiv -S^{-1/3}\Delta_s'$ is the (normalized to $S^{-1/3}$) conventional tearing stability index of the (stable) m, n mode, $P \equiv \tau_R/\tau_V = \mu_0\mu_i(r_s)/[\eta(r_s)\rho_m(r_s)]$ is the magnetic Prandtl number at the resonant surface, and the perpendicular viscous timescale is given by $\tau_V = r_s^2\rho_m(r_s)/\mu_i(r_s)$, where $\mu_i(r_s)$ is the (dimensional) viscosity. Since $S \gg 1$ and $P \geq 1$ in a high temperature tokamak plasma and a tearing-stable m, n mode is assumed, $|\Delta_s'| \sim \mathcal{O}(1)$, $\alpha \ll 1$, and thus to a good approximation we may neglect α in the above torque balance equation. The error-field penetration threshold corresponds to the critical error-field amplitude above which torque balance is lost, *i.e.*, where the approximated torque balance equation has no solution [4]. It follows that

$$\left|\frac{b_{r,nm}^{\text{vac}}}{B_\phi}\right|_{\text{crit}}^2 = \max\left\{\frac{P}{\kappa S}\frac{(Q_*^{NC} - Q)\left|\hat{\Delta}(Q)\right|^2}{\text{Im}\{\hat{\Delta}(Q)\}}\right\}, \tag{15}$$

where the maximum is obtained by varying Q.

7. Relevant Layer Regimes

Recalling $S \gg 1$, $P \geq 1$, and inspecting Sect. III G of [5], it follows that the three most likely error-field response regimes are the *1st Visco-Resistive* (VRi) regime, the *1st Semi-Collisional* (SCi) regime, and the *1st Hall-Resistive* (HRi) regime—see Table 1 below. The VRi regime holds when $D^2\,P^{1/3} < Q$, the SCi regime holds when $\beta^{1/2}\,D < \sqrt{2}\,Q < \sqrt{2}\,D^2\,P^{1/3}$, and the HRi regime holds when $\sqrt{2}\,Q < \beta^{1/2}\,D$. Here, $\beta = 10\,\mu_0 P_0/(3B_0^2)$ is the toroidal beta, where P_0 is the equilibrium plasma pressure, and $D = S^{1/3}\,\rho_s(r_s)/r_s$. The quantity $\rho_s(r_s)$ is the ion Larmor radius at the resonant surface, calculated using the electron temperature.

Table 1: *Tokamak-relevant linear drift-MHD response regimes [5] for a static error-field. Abbreviations indicate the different response regimes: 1st Hall-Resistive [HRi]; 1st Semi-Collisional [SCi]; 1st Visco-Resistive [VRi]. Here,* $\hat{\Delta} = S^{-1/3}\,\Delta$, $Q = S^{1/3}\,\omega\tau_H$, $Q_{i,e} = -S^{1/3}\,\omega_{*i,e}\,\tau_H$, $D = S^{1/3}\,\rho_s(r_s)/r_s$, *and* $P = \tau_R/\tau_V$. *Here* $\rho_s(r_s)$ *is the ion Larmor radius at the resonant surface, calculated using the electron temperature. Finally,* $\tau = T_i/T_e$ *is the ratio of the ion and electron temperatures. Note the numerical coefficient of the HRi regime differs from that given in [5] owing to a factor of 2 difference in the definition of* β.

Abbreviation	*Response*
HRi	$\hat{\Delta} = 1.786\,[\mathrm{i}\,(Q - Q_e)]\,\beta^{1/4}\,D^{-1/2}\,(1+\tau)^{-1/4}$
SCi	$\hat{\Delta} = 3.142\,[\mathrm{i}\,(Q - Q_i)]^{1/2}\,[\mathrm{i}\,(Q - Q_e)]\,D^{-1}\,(1+\tau)^{-1/2}$
VRi	$\hat{\Delta} = 2.104\,[\mathrm{i}\,(Q - Q_i)]^{1/6}\,[\mathrm{i}\,(Q - Q_e)]^{5/6}\,P^{1/6}$

Using a Padé approximation valid for all values of Γ_s in each of the three layer regimes VRi, SCi, and HRi respectively, we find the error-field penetration threshold in each regime:

$$\left|\frac{b^{vac}_{r,nm}}{B_\phi}\right|^2_{\text{crit,VRi}} \simeq \frac{[s(r_s)]^2}{\lambda}\frac{P^{7/6}}{S^{1/3}}(\omega_*\tau_H)^2\left[\frac{1+\chi+\chi^2}{1+\chi}\right], \tag{16}$$

$$\left|\frac{b^{\rm vac}_{r,nm}}{B_\phi}\right|^2_{\text{crit,SCi}} \simeq \frac{[s(r_s)]^2 r_s}{\lambda R_0}\frac{P(\omega_*\tau_H)^{5/2}}{\rho_* S^{1/2}}\left[\frac{1+\gamma+\gamma^2}{1+\gamma}\right], \tag{17}$$

$$\left|\frac{b^{vac}_{r,nm}}{B_\phi}\right|^2_{\text{crit,HRi}} \simeq \frac{[s(r_s)]^2\beta^{1/4}}{\rho_*^{1/2}\lambda}\left(\frac{r_s}{R_0}\right)^{1/2}\frac{P}{S^{1/2}}(\omega_*\tau_H)^2\left[\frac{1+\chi+\chi^2}{1+\chi}\right], \tag{18}$$

where $\lambda = 2\int_{r_s}^a [\mu(r_s)/\mu(r)]\,(dr/r)$, $\gamma = [R_0 m/(r_s n)]^{5/2}\lambda\Gamma_s$, and $\chi = [R_0 m/(r_s n)]^2\lambda\Gamma_s$. For simplicity, we have assumed $T_i \simeq T_e$ which implies $\omega_{*,i}$ scales as $\omega_{*,e} \equiv \omega_*$, where $\omega_{*,e}$ is the (dimensional) electron diamagnetic flow frequency at the resonant surface.

8. Tokamak Scaling Study

As an application of this theory, consider a class of ohmically heated tokamak plasmas in which the aspect ratio, R_0/a, and the equilibrium profiles are held *fixed*. By definition, $\omega_*\,\tau_H \propto T_e\,\sqrt{n_e}/\left(R_0\,B_\phi^2\right)$, $S \propto B_\phi\,T_e^{3/2}\,R_0/\sqrt{n_e}$, $\beta \propto n_e T_e/B_\phi^2$, $\rho_* \propto T_e^{1/2}/(R_0\,B_\phi)$, and $P \propto R_0^2\,T_e^{3/2}/\tau_V$. Finally, in the low collisionality ($1/\nu$) NTV regime $\nu_{\|} = \omega_{ti}^2/\nu_i \propto T_e^{5/2}/(R_0^2 n_e)$. Thus, in the NTV dominated limit $\Gamma_s \gg 1$, (16)-(18) reduce to

$$\left|\frac{b_{r,nm}^{\text{vac}}}{B_\phi}\right|_{\text{crit,VRi}} \sim n_e^{2/3} B_\phi^{-13/3} R_0^{-1} T_e^{9/2} \tau_V^{-2/3}\, \sigma, \tag{19}$$

$$\left|\frac{b_{r,nm}^{\text{vac}}}{B_\phi}\right|_{\text{crit,SCi-HRi}} \sim n_e B_\phi^{-9/2} R_0^{-1} T_e^{4} \tau_V^{-1/2}\, \sigma, \tag{20}$$

$$\sigma = \sqrt{\sum_{(n',m')\neq(0,0)} \left|n' b_{n'm'}/b_{r,nm}^{\text{vac}}\right|^2 W_{n'm'}}. \tag{21}$$

(Under the substitutions above, the HRi and SCi regimes scale identically.) Here, σ is the ratio of the "effective" non-resonant to resonant error field at the resonant surface. From now on, we assume the non-resonant and resonant error-field components scale similarly, i.e. $\sigma \propto$ constant. Ohmic power balance allows us to eliminate T_e in favor of the energy confinement time τ_E

$$T_e = \left(\frac{\tau_E}{n_e}\right)^{2/5} \left(\frac{B_\phi}{R_0}\right)^{4/5}, \tag{22}$$

which further reduces the penetration thresholds to

$$\left|\frac{b_{r,nm}^{\text{vac}}}{B_\phi}\right|_{\text{crit,VRi}} \sim n_e^{2/3} B_\phi^{-11/15} R_0^{-4.6} \left(\frac{\tau_E}{n_e}\right)^{9/5} \tau_V^{-2/3}, \tag{23}$$

$$\left|\frac{b_{r,nm}^{\text{vac}}}{B_\phi}\right|_{\text{crit,SCi-HRi}} \sim n_e B_\phi^{-1.3} R_0^{-4.2} \left(\frac{\tau_E}{n_e}\right)^{8/5} \tau_V^{-1/2}. \tag{24}$$

Experiments on JET, DIII-D, and Alcator C-Mod find the error-field penetration threshold scales approximately linearly with electron density and inversely with toroidal magnetic field strength [1]. Experimental scaling with major radius is not directly measured, but inferred from the observed scalings with electron density and toroidal field strength via dimensionless scaling arguments [1, 19]. To reduce (23) and (24) further requires a detailed knowledge of the perpendicular momentum confinement time τ_V in low-density ohmic discharges. Assuming a neo-Alcator energy confinement scaling $\tau_E \propto n_e R_0^{13/4}$ (albeit slightly modified in the R_0 dependence to satisfy dimensionless arguments), and either Bohm or gyro-Bohm diffusion, we find the theoretical penetration thresholds scale as

$$\left|\frac{b_{r,nm}^{\text{vac}}}{B_\phi}\right|_{\text{crit,VRi}}^{Bohm} \sim n_e^{2/3} B_\phi^{-13/15} R_0^{1/4}, \tag{25}$$

$$\left|\frac{b_{r,nm}^{\text{vac}}}{B_\phi}\right|_{\text{crit,SCi-HRi}}^{Bohm} \sim n_e B_\phi^{-1.4} R_0^{1/4}, \tag{26}$$

$$\left|\frac{b_{r,nm}^{\text{vac}}}{B_\phi}\right|_{\text{crit,VRi}}^{g-Bohm} \sim n_e^{2/3} B_\phi^{-19/15} R_0^{-1/4}, \tag{27}$$

$$\left|\frac{b_{r,nm}^{\text{vac}}}{B_\phi}\right|_{\text{crit,SCi-HRi}}^{g-Bohm} \sim n_e B_\phi^{-1.7} R_0^{-1/8}. \tag{28}$$

Suppose instead, that perpendicular viscosity is dominated by collisional paleoclassical electron diffusion [20], which predicts $T_e \propto R_0^{1/3} B_\phi^{2/3}$ and $\tau_V \propto R_0^{5/2} B_\phi$. Under these assumptions (19) and (20) reduce to

$$\left|\frac{b^{\rm vac}_{r,nm}}{B_\phi}\right|^{paleo}_{\rm crit,VRi} \sim n_e^{2/3} B_\phi^{-2} R_0^{-7/6}, \tag{29}$$

$$\left|\frac{b^{\rm vac}_{r,nm}}{B_\phi}\right|^{paleo}_{\rm crit,SCi\text{-}HRi} \sim n_e B_\phi^{-7/3} R_0^{-11/12}. \tag{30}$$

While there is considerable uncertainty in these estimates for energy confinement and perpendicular viscous times, we find they all predict nearly linear density dependence for the penetration threshold—a result that is universally observed in empirical scaling studies [1]. Furthermore, the theoretical scaling of the penetration threshold with engineering parameters is of the general form $[b_r(r_s)/B_\phi]_{\rm crit} \sim n_e^{\alpha_n} B_\phi^{\alpha_B} R_0^{\alpha_R}$, where $\alpha_R = 2\,\alpha_n + 1.25\,\alpha_B$.

9. Conclusions

In the limit $1 \ll \Gamma_s \ll 1/\delta$ neoclassical toroidal viscosity [NTV] enhances perpendicular viscosity near (but not within) the resonant layer, thus increasing the critical resonant error-field strength required for locking. The new penetration thresholds all have two novel features: 1) a stronger dependence on electron density than previously predicted [5] (a result in qualitative agreement with empirical scaling studies [1] if T_e and τ_V do not depend strongly on n_e); 2) a dependence on the ratio, σ, between the non-resonant and resonant error-field components, a feature that could be tested in current tokamaks to determine the relevance of neoclassical toroidal viscosity [NTV] in ohmic discharges. This work was supported by the U.S. Department of Energy under Grant Nos. DE-FG02-86ER53218 and DE-FG02-92ER54139.

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