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ECRH and its effects on neoclassical transport in stellarators

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Abstract

Energetic electron populations can substantially modify the neoclassical transport properties in stellarators. A model accounting for this change in transport is developed assuming the presence of electron cyclotron resonance heating (ECRH). The quasilinear diffusion coefficient for second harmonic X-mode ECRH is developed for a bumpy stellarator. Care is taken in accounting for the pitch-angle dependence of the quasilinear diffusion coefficient since application to experiments with narrow resonance zones is of interest. Weakly relativistic effects are considered through the mass effect on the cyclotron frequency. For trapped particles in a three dimensional configuration, collisionless loss zones exist in velocity space. Radio-frequency (rf) waves accelerate trapped electrons into the direct loss zone in bumpy stellarators and produce a direct loss flux. An analytic expression for this loss flux is derived; it is proportional to the rf field strength and the value of the zeroth order distribution function at the minimum speed for collisionless loss. The direct loss flux of electrons is another source of a non-ambipolar particle flux in bumpy stellarators. This additional non-ambipolar flux modifies the ambipolarity equation which generally has multiple roots for the radial electric field. An electron root (large positive E_r) is easily obtained if the electrons are in the $1/\nu$ regime and the ions are in the ν regime.

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I. INTRODUCTION

Trapped particles in a symmetric magnetic confinement system are confined. In contrast, helically trapped particles in systems without a symmetry such as conventional stellarators are poorly confined in the low collisionality regime [1]. Trapped particle orbits deviate significantly from magnetic surfaces. Therefore, neoclassical transport becomes poor in the low collisionality regime. However, the radial excursions and neoclassical transport are reduced by the poloidal $\vec{E} \times \vec{B}$ drift [1]. Therefore, a large positive radial electric field is required to reduce the neoclassical transport in the low collisionality regime. In this work, we address the role a large energetic electron population can have in determining the radial electric field. In particular, we put forth a model that accounts for the change in the self-consistent neoclassical transport when a poorly confined electron tail population is created in a three-dimensional configuration.

The radial electric field E_r is determined self-consistently from the ambipolarity condition [2]. In a stellarator, the cross-field particle fluxes are not intrinsically ambipolar [3] as in a tokamak. If all species are in the $1/\nu$ regime, the electric field is typically negative because the ions leave the device more rapidly than the electrons [4]. If all species are in the ν (or $\sqrt{\nu}$) regime, the electric field is weakly positive because the electrons leave the device more rapidly than the ions. However, the ambipolarity equation can have more than one solution when the ions are in the ν regime and the electrons are in the $1/\nu$ regime since the ion flux is a nonlinear function of the electric field. To obtain a large radial electric field, a large electron particle flux is required.

Due to the large radial excursions of trapped particles [5], particles trapped in the helical magnetic wells can leave the device directly when the collision frequency is low enough. For energetic particles generated by electron cyclotron resonance heating (ECRH), the collision length can easily be longer than the time for a collisonless orbit loss. Direct losses of the energetic electrons generated by ECRH provides an additional non-ambipolar flux. The additional non-ambipolar flux by the direct loss of trapped particles raises the possibility that the ambipolarity equation has a large positive root. This feature has been observed in recent stellarator experiments on LHD [6, 7] and W7-AS [8–10]. In this paper, we develop the quasi-linear diffusion coefficient for second harmonic X-mode ECRH in a bumpy stellarator. We then use an analytic calculation to quantify the direct loss flux generated by ECRH.

In section II, we develop a quasilinear diffusion coefficient for the second harmonic X-mode ECRH. In section III, we quantify the direct loss particle flux generated by ECRH in bumpy stellarators. In section IV, we discuss the ambipolarity condition when the direct loss flux is included. Conclusions of this paper are summarized in section V.

II. QUASI-LINEAR DIFFUSION COEFFICIENT FOR ECRH

When particles experience the wave electric field at their cyclotron frequency or a multiple of it in the laboratory frame via a Doppler shift, cyclotron heating occurs. In stellarators, a significant fraction of particles are trapped as in tokamaks. Trapped particles are bouncing back and forth, and gain energy from the waves more frequently than do the passing particles. Moreover, trapped particles whose turning points coincide with the resonance region spend a considerably longer time inside the resonance region, and are most strongly heated there. Thus, the trapped particle distribution is expected to deviate most strongly from a Maxwellian distribution.

A. Analytic description

In this section and throughout the rest of the paper, we will use a "bumpy stellarator"model magnetic field strength given by

$$\frac{B}{B_0} \simeq 1 - \epsilon_h \cos(M\theta - N\phi) - \epsilon_m \cos N\phi, \tag{1}$$

where $\epsilon_h = B_{M,N}/B_{0,0}$, $\epsilon_h = B_{0,N}/B_{0,0}$ and M and N are the poloidal and toroidal mode numbers. In the limit $\epsilon_m = 0$, we have a magnetic field spectrum that describes a helically symmetric magnetic field. With $\epsilon_m \neq 0$, three dimensional effects produce collisonless direct orbit losses.

The cyclotron heating process is described by using unperturbed particle orbits [11]. The particles pass through the resonance region following a magnetic field line. On each passage through the resonance region the particles receive a kick in perpendicular speed $v_\perp$. We assume that the heating is stochastic because the collision rate is usually sufficient to randomize the gyrophase between resonant kicks and thereby destroy superadiabaticity. The quasi-linear diffusion coefficient is drived for the extraordinary wave at the second harmonic cyclotron resonance with electrons.

The Lorentz equation for electrons is used to calculate the energy gained per passage through the resonance:

$$\frac{d(\gamma m_0 \vec{v})}{dt} = -e(\vec{v} \times \vec{B} + \vec{E}). \tag{2}$$

The rate of energy ($\mathcal{E}$) change induced by an external electric field is

$$\frac{d\mathcal{E}}{dt} = \frac{d(\gamma m_0 c^2)}{dt} = -\; e\; \vec{v}_\perp \cdot \vec{E}, \tag{3}$$

where

$$\gamma = \left(1 - \frac{\vec{v} \cdot \vec{v}}{c^2}\right)^{-1/2} \simeq 1 + \frac{v^2}{2c^2} \tag{4}$$

in the weakly relativistic limit and

$$\vec{E} = E_x \hat{x} \sin(kx - \omega t) + E_y \hat{y} \cos(kx - \omega t). \tag{5}$$

Using $v_x = -\; v_\perp \sin\psi$, $v_y = v_\perp \cos\psi$, $x = \rho_L \cos\psi$, $\rho_L \equiv v_\perp/\Omega$, $\Omega \equiv eB/m$ and inserting Eq. (5) into Eq. (3), we obtain

$$\frac{d\gamma}{dt} = \frac{ev_\perp}{m_0 c^2}\left[E_x \sin\psi \sin(k\rho_L \cos\psi - \omega t) - E_y \cos\psi \cos(k\rho_L \cos\psi - \omega t)\right]. \tag{6}$$

Equation (6) is expanded in Bessel functions. Retaining only the secular term, we obtain

$$\frac{d\gamma}{dt} \approx \frac{ev_\perp E J_1(k\rho_L)}{m_0 c^2} \sin(2\psi - \omega t), \tag{7}$$

where $E = E_x + E_y$. For ECRH microwaves $k\rho_L \sim (k_L v_\perp/\omega) \sim v_\perp/c \ll 1$, so we can assume that $k\rho_L \ll 1$. When $k\rho_L \ll 1$, we expand $J_1(k\rho_L) \approx k\rho_L/2$. Then, Eq. (7) becomes

$$\frac{d\gamma}{dt} \approx \frac{ev_\perp^2 Ek}{2m_0 \Omega_{ce} c^2} \sin(2\psi - \omega t). \tag{8}$$

The average change in perpendicular energy per transit $\Delta W_\perp$ is

$$\Delta W_\perp \approx \Big| \int_{-\infty}^{\infty} dt \frac{ev_\perp^2 Ek}{2\Omega_{ce}} \sin(2\psi - \omega t) \Big|. \tag{9}$$

A linear approximation is used to model the variation of the magnetic field strength along a field line. The gyrofrequency along the field line around the resonance zone is written as

$$\Omega(t) = \Omega_0 + v_\parallel \Omega' t, \tag{10}$$

where $\Omega' = \partial\Omega/\partial z$. Then, the phase between the particles and the waves becomes

$$\psi \equiv \int^t dt' \Omega(t') = \psi_0 + \Omega_0 t + \frac{1}{2} v_\parallel \Omega' t^2, \tag{11}$$

where ψ_0 is the initial phase and Ω_0 is the electron gyrofrequency at the resonance position. The relativistic correction to the gyrofrequency is also considered. Equation (11) thus becomes

$$\psi \equiv \int^t dt'\Omega(t') = \psi_0 \left(1 - \frac{v^2}{2c^2}\right) + \Omega_0 t + \frac{1}{2} v_\parallel \Omega' t^2. \tag{12}$$

The spatial inhomogeneity of the ECRH wave field is expressed as a Gaussian profile. Particles passing the injected microwave beam absorb energy from the waves as long as the resonance condition is satisfied. For simplicity, the wave beam intensity is expressed along the magnetic field line direction as

$$E(z) = E_0 \exp\left(-\frac{z^2}{2L_b^2}\right), \tag{13}$$

where L_b is the effective ECRH beam width.

The integration variable in Eq. (9) is changed from t to z using $dt = dz/v_\parallel$. Then, we rewrite Eq. (9) in the form

$$\Delta W_\perp \approx \left| \int_{-\infty}^{\infty} \frac{dz}{v_\parallel} \frac{ev_\perp^2 E_0 k}{2\Omega_{ce}} \exp\left[-\frac{z^2}{2L_b^2} + i\left(2\psi_0 - \frac{v^2}{2c^2}\Omega_0 \frac{z}{v_\parallel} + \Omega' \frac{z^2}{v_\parallel}\right)\right]\right|. \tag{14}$$

Because the heating is assumed to be stochastic, we average the energy increase over the initial phase ψ_0 between the ECRH microwaves and electrons. Then, the perpendicular energy increase per pass through the resonance becomes

$$\Delta W_\perp \approx \left| \frac{\sqrt{\pi}}{2} \frac{ekv_\perp^2 E_0}{\Omega_{ce}} \frac{1}{v_\parallel \left(1/4L_b^4 + \Omega'^2/v_\parallel^2\right)} \exp\left[-\frac{v^4\Omega_0^2}{8L_b^2 c^4 v_\parallel^2} \frac{1}{1/4L_b^4 + \Omega'^2/v_\parallel^2}\right]\right|. \tag{15}$$

Taking $\Delta W_\perp \simeq mv_\perp \Delta v_\perp$, we find the average perpendicular speed increase to be

$$\Delta v_\perp = \frac{\Delta W_\perp}{mv_\perp} = \left| \frac{\sqrt{\pi}}{2} \frac{kv_\perp E_0}{B_0} \frac{1}{v_\parallel \left(1/4L_b^4 + \Omega'^2/v_\parallel^2\right)^{1/4}} \exp\left[-\frac{v^4\Omega_0^2}{8L_b^2 c^4 v_\parallel^2} \frac{1}{1/4L_b^4 + \Omega'^2/v_\parallel^2}\right]\right|. \tag{16}$$

We assume that the wave beam width is wide compared to the width of the resonance region ($1/L_b^2 \ll \Omega'/v_\parallel$). Then, Eq.(16) becomes

$$\Delta v_\perp = \frac{\Delta W_\perp}{mv_\perp} = \left| \frac{\sqrt{\pi}}{2} \frac{kv_\perp E_0}{B_0} \frac{1}{v_\parallel^{1/2}\Omega'^{1/2}} \exp\left[-\frac{1}{8L_b^2}\left(\frac{\Omega_0}{\Omega'}\right)^2 \frac{v^4}{c^4}\right]\right|. \tag{17}$$

Thus, the quasi-linear diffusion coefficient is

$$D = \frac{(\Delta v_\perp)^2}{2\Delta t} = \frac{\pi}{8\Delta t} \frac{k^2 v_\perp^2 E_0^2}{B_0^2} \frac{1}{v_\parallel \Omega'} \exp\left[-\frac{1}{4L_b^2}\left(\frac{\Omega_0}{\Omega'}\right)^2 \frac{v^4}{c^4}\right]. \tag{18}$$

For trapped particles, Δt is half of the period τ of the bouncing motion, which is given by

$$\tau = \oint \frac{dl}{v_\parallel} = \frac{\partial J}{\partial W} = \frac{4R}{N}\left(\frac{m}{\mu_m \epsilon_H B_0}\right)^{1/2} K(\kappa), \tag{19}$$

where N is the number of field periods in the magnetic field, R is the major radius, μ_m is the magnetic moment, $\epsilon_H = (\epsilon_h^2 + \epsilon_m^2)^{1/2}$ and the pitch-angle variable κ is defined by

$$\kappa^2 = \frac{W - \mu_m B_0(1-\epsilon_H)}{2\epsilon_H \mu_m B_0} \approx \frac{1}{2\epsilon_H}\frac{v_\parallel^2}{v_\perp^2}.$$

Finally, the bounce-averaged quasilinear diffusion coefficient for second harmonic X-mode ECRH in a bumpy cylinder model with the Gaussian wave field mode in Eq. (13) is given by

$$D = \frac{(\Delta v_\perp)^2}{\tau} \simeq \frac{\pi N}{16\sqrt{2}R}\frac{k^2 v_\perp^2 E_0^2}{B_0^2}\frac{v_{\perp 0}}{v_{\parallel res}}\frac{\epsilon_H^{1/2}}{\Omega' K(\kappa)} \exp\left[-\frac{1}{4L_b^2}\left(\frac{\Omega_0}{\Omega'}\right)^2 \frac{v^4}{c^4}\right]. \tag{20}$$

B. Pitch angle dependence

Since trapped particles stay near the resonance zone longer, they gain more energy than particles that pass through the resonance zone quickly. Trapped particles that are turning within the beam receive the maximum heating. Therefore, the pitch-angle dependence of the diffusion coefficient is important. In Eq. (20) the diffusion coefficient D has a pitch angle dependence through $v_{\parallel res}$ and $K(\kappa)$. We can rewrite $v_{\parallel res}/v_{\perp 0}$ as

$$\frac{v_{\parallel res}^2}{v_{\perp 0}^2} = \frac{v^2 - v_{\perp res}^2}{v_{\perp 0}^2} = \frac{v_{\parallel 0}^2 + v_{\perp 0}^2}{v_{\perp 0}^2} - \frac{B_{res}}{B_0} = \frac{v_{\parallel 0}^2}{v_{\perp 0}^2} - \frac{B_{res} - B_0}{B_0} = 2\epsilon_H(\kappa^2 - \kappa_0^2), \tag{21}$$

where

$$\kappa_0^2 = \frac{B_{res} - B_0}{2\epsilon_H B_0}.$$

Here κ_0 is the value for trapped particles whose turning points coincide with the resonance region. Thus, we rewrite Eq. (20) as

$$D = \frac{(\Delta v_\perp)^2}{\tau} = \frac{\pi N}{32R}\frac{k^2 v_\perp^2 E_0^2}{B_0^2 \Omega'}\frac{1}{(\kappa^2 - \kappa_0^2)^{1/2} K(\kappa)} \exp\left[-\frac{1}{4L_b^2}\left(\frac{\Omega_0}{\Omega'}\right)^2 \frac{v^4}{c^4}\right]. \tag{22}$$

However, Eq. (22) is not valid for trapped electrons turning inside the beam width since the parallel velocity $v_\parallel$ becomes zero within the beam. Generally, the time that electrons spend within the beam is $L_b/v_\parallel$. Trapped particles turning inside the beam width are decelerated to $v_\parallel = 0$ and accelerated backward by the force arising from $\nabla_\parallel B$, which is

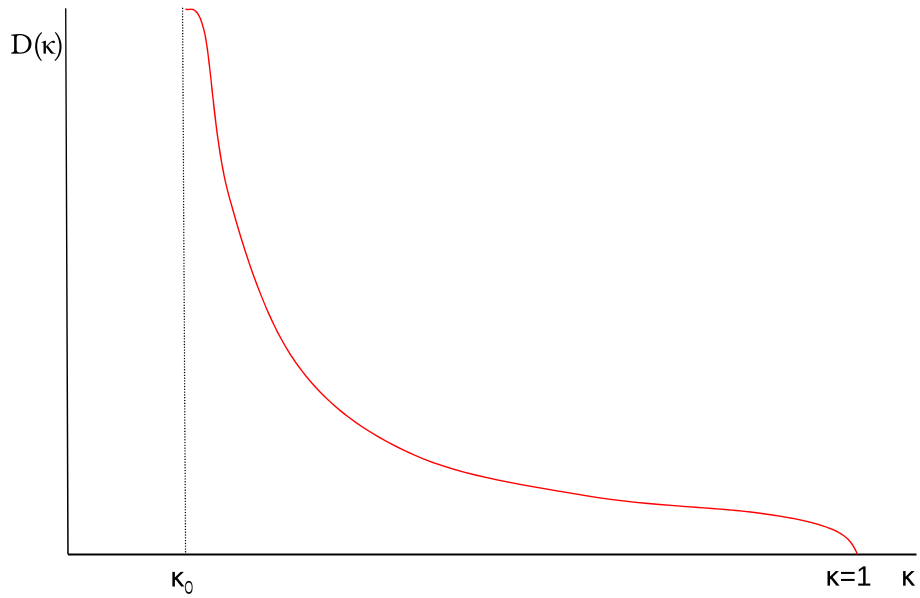

FIG. 1: The quasilinear diffusion coefficient with the second harmonic X mode ECRH versus the pitch angle variable κ. The diffusion coefficient is localized around κ_0. Here, $\kappa = 1$ corresponds to the trapped-passing boundary.

expressed as $F_{\parallel} = -\mu_m \partial B/\partial z$. The time for those localized trapped electrons to travel through the resonance is

$$\tau_{res} \approx 2\sqrt{\frac{2L_b}{a_{res}}} \approx \sqrt{\frac{2L_b}{(v_{\perp 0}^2/2B_0)\partial B/\partial z}} = \frac{4}{v_{\perp 0}}\sqrt{L_b\frac{\Omega_0}{\Omega'}} \sim \frac{4}{v_{\perp 0}}\sqrt{\frac{L_b L}{2\epsilon_H}}, \tag{23}$$

where L is the magnetic well width. The electrons whose κ values are smaller than κ_0 do not gain energy from the ECRH microwaves since they turn before they experience the ECRH microwaves.

Thus, the full expression of the quasi-linear velocity-space diffusion coefficient becomes

$$D \approx \begin{cases} \dfrac{\pi N}{32R}\dfrac{k^2 v_\perp^2 E_0^2}{B_0^2\Omega'}\dfrac{1}{(\kappa^2-\kappa_0^2)^{1/2}K(\kappa)}\exp\left[-\dfrac{1}{4L_b^2}\left(\dfrac{\Omega_0}{\Omega'}\right)^2\dfrac{v^4}{c^4}\right], & \kappa > \kappa_0, \\ \dfrac{\pi N}{8R}\dfrac{k^2 v_\perp^2 E_0^2}{B_0^2\Omega'}\dfrac{L_b}{L}\dfrac{1}{K(\kappa_0)}\exp\left[-\dfrac{1}{4L_b^2}\left(\dfrac{\Omega_0}{\Omega'}\right)^2\dfrac{v^4}{c^4}\right], & \kappa \approx \kappa_0, \\ 0, & \kappa < \kappa_0. \end{cases} \tag{24}$$

Figure 1 shows that this diffusion coefficient has its maximum value at κ_0, and is highly peaked near this point.

If particles never leave the resonance zone, the gyrophase remains correlated with the waves. In this case, quasilinear theory is not adequate and a nonlinear interaction between particles and the waves occurs. Due to relativistic effects, particles detune from the resonance as the energy increases. The relativistic phase slippage pushes electrons away from the resonance so that the energy decreases until electrons recover the resonance. Finally, an energy excursion occurs, repeatedly detuning and retuning to the resonance [12].

To estimate the nonlinear interaction between the waves and particles, consider particles bouncing back and forth inside the microwave beam with $L_{bounce} < L_b$. A model magnetic field configuration $B = B_0(1 - \epsilon_H \cos\chi)$ is used where χ is a phase angle along the magnetic field line. At the turning points,

$$B_{turn} = B_0(1 - \epsilon_H \cos\chi_{turn}) = \frac{B_{min}}{\sin^2\vartheta} = \frac{B_0(1-\epsilon_H)}{\sin^2\vartheta},$$

where ϑ is the pitch angle of trapped electrons. Therefore, L_{bounce} is given as

$$L_{bounce} \sim 2R\chi_0 = 2R \arccos\left[\frac{\epsilon_H - \cos^2\vartheta}{\epsilon_H \sin^2\vartheta}\right].$$

For typical parameters of interest ($L_b = 5$ cm, $\epsilon_H = 0.1$, $R = 1$ m) the pitch angle needs to be between 89.67 and 90 degrees. For the same parameters, the pitch-angle of trapped particles varies from 65 to 90 degrees. Thus, only a very small fraction of trapped particles experience a nonlinear interaction with the microwaves. Moreover, for a significant nonlinear interaction between the microwaves and trapped particles, the microwave beam must be focused on the bottom of the magnetic well. Thus, nonlinear interactions typically do not play a significant role in the heating process in bumpy stellarators and will be neglected in the following calculation.

III. DIRECT LOSS GENERATED BY ECRH

A. Condition for direct loss

When the energy of trapped electrons is high enough to be collisionless, electrons drift out of the device without collisions along the $B_{min} = constant$ surfaces [5, 13, 14]. The minimum speed for direct loss is given by the condition $\dot{r}/a > \nu_{eff}$ where $\nu_{eff} = \nu_{ei}/\epsilon_H$. In bumpy stellarators, the radial drift speed of the particle guiding center is

$$\dot{r} = \frac{\mathcal{E}_\perp}{eBr}\frac{\partial\epsilon_H}{\partial\theta}\left(\frac{2E(\kappa)}{K(\kappa)} - 1\right) \simeq \frac{\mathcal{E}_\perp \epsilon_m}{2eB\epsilon_H}, \qquad (25)$$

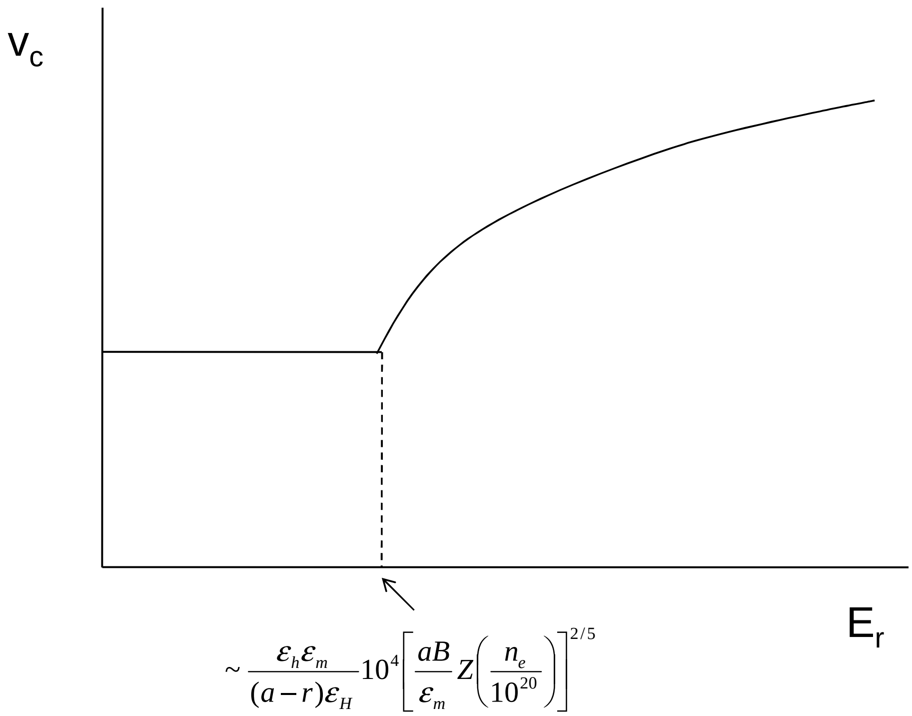

FIG. 2: The minimum speed of trapped particle for direct loss versus the radial electric field E_r. The radial electric field plays a role when the $\vec{E} \times \vec{B}$ rotation is dominating.

where $E(\kappa)$ and $K(\kappa)$ are elliptic integrals and $\mathcal{E}_\perp = mv_\perp^2/2$. The condition for direct loss in a bumpy stellarator is

$$\frac{\mathcal{E}_\perp \epsilon_m}{2aB\epsilon_H} > \frac{\nu_{ei}}{\epsilon_H} \simeq 6.3 \times 10^9 Z\ \mathcal{E}_e^{-3/2} \left(\frac{n_e}{10^{20}} \right) / \epsilon_H, \tag{26}$$

where n is the electron density in m^{-3}, Z is the ratio of the ion charge to electron charge and $\mathcal{E}_\perp$ and $\mathcal{E}_e$ are in eV. Since $\mathcal{E}_\perp \simeq \mathcal{E}_e$ for trapped particles, the minimum energy $\mathcal{E}_c$ for direct loss is estimated as

$$\mathcal{E}_c = \frac{1}{2} m v_c^2 \simeq 10^4 \left[\frac{aB}{\epsilon_m} Z \left(\frac{n_e}{10^{20}} \right) \right]^{2/5}, \tag{27}$$

where v_c is the minimum speed of trapped electrons that escape the plasmas without collisons.

When the radial electric field is dominant, the radial excursions of trapped particle orbits are reduced. The displacement of the orbit is given by

$$\delta r \simeq \frac{\mathcal{E}_\perp \epsilon_m}{B\epsilon_H} \left(\frac{2E(\kappa)}{K(\kappa)} - 1 \right) / \omega_E, \tag{28}$$

where $\omega_E = E_r/Br$. When the displacement δr is bigger than the distance between the magnetic surface and the plasma boundary, trapped particles are not confined, i.e., for

$\delta r > a - r$. The minimum energy for direct loss is estimated as

$$\mathcal{E}_c \simeq \frac{\epsilon_H}{\epsilon_m \epsilon_h}(a-r)E_r. \tag{29}$$

From Eqs. (27) and (29), the minimum speed v_c for direct loss is written as

$$v_c \simeq \begin{cases} 148\left(\frac{e}{m}\right)^{1/2}\left[\frac{aB}{\epsilon_m}Z\left(\frac{n_e}{10^{20}}\right)\right]^{1/5}, \; E_r < \frac{\epsilon_h\epsilon_m}{(a-r)\epsilon_H}10^4\left[\frac{aB}{\epsilon_m}Z\left(\frac{n_e}{10^{20}}\right)\right]^{2/5}, \\ \\ \left[\frac{2e\epsilon_H}{m\epsilon_m\epsilon_h}(a-r)E_r\right]^{1/2}, \qquad E_r > \frac{\epsilon_h\epsilon_m}{(a-r)\epsilon_H}10^4\left[\frac{aB}{\epsilon_m}Z\left(\frac{n_e}{10^{20}}\right)\right]^{2/5}. \end{cases} \tag{30}$$

Equation (30) is plotted in Fig. 2; it shows the dependence of the minimum speed on E_r. Once trapped electrons have an energy greater than $\mathcal{E}_c$, they leave the magnetic surface directly since the radial drift motion of the trapped electron is faster than collisional scattering in pitch-angle out of that region. Therefore, a continuous source of energetic electrons is needed to sustain the direct loss in steady state.

B. Direct loss rate

ECRH accelerates electrons beyond a minimum energy level. A Fokker-Planck model is used to describe the generation of energetic electrons and direct loss rate:

$$\frac{\partial f}{\partial t} = -\nabla_{\mathbf{v}}\cdot\left(\left\langle\frac{\Delta\mathbf{v}}{\Delta t}\right\rangle f\right) + \frac{1}{2}\nabla_{\mathbf{v}}\cdot\left[\nabla_{\mathbf{v}}\cdot\left(\left\langle\frac{\Delta\mathbf{v}\Delta\mathbf{v}}{\Delta t}\right\rangle f\right)\right] = Q(f) + C(f). \tag{31}$$

Trapped electrons below the critical energy are accelerated above the critical energy by obtaining energy from the rf waves. The direct loss rate is calculated by estimating the net particle flux in velocity space across the $v = v_c$ surface caused by ECRH (Fig. 3):

$$\dot{n} \approx \int_{V_{loss}} d^3v\, Q(f) \approx \int d^3v \left\{ -\nabla_{\mathbf{v}}\cdot\left(\left\langle\frac{\Delta\mathbf{v}}{\Delta t}\right\rangle f\right) + \frac{1}{2}\nabla_{\mathbf{v}}\cdot\left[\nabla_{\mathbf{v}}\cdot\left(\left\langle\frac{\Delta\mathbf{v}\Delta\mathbf{v}}{\Delta t}\right\rangle f\right)\right]\right\}. \tag{32}$$

Here the $C(f)$ Coulomb operator is ignored since electrons tend to be decelerated by collisions rather than accelerated by them. Using Gauss' theorem, Eq. (32) becomes

$$\dot{n} \approx -\int_{S_{loss}} d^2v\, \mathbf{n}\cdot\left[\left\langle\frac{\Delta\mathbf{v}}{\Delta t}\right\rangle f + \nabla_{\mathbf{v}}\cdot\left(\left\langle\frac{\Delta\mathbf{v}\Delta\mathbf{v}}{\Delta t}\right\rangle f\right)\right], \tag{33}$$

where $\mathbf{n}$ is a normal vector of the surface S_{loss}. Assuming that the contribution of the pitch-angle diffusion term is negligible, Eq. (33) becomes

$$\dot{n} \approx -\int_{S_{loss}} d^2v\, \mathbf{n}\cdot\left\langle\frac{\Delta\mathbf{v}}{\Delta t}\right\rangle f. \tag{34}$$

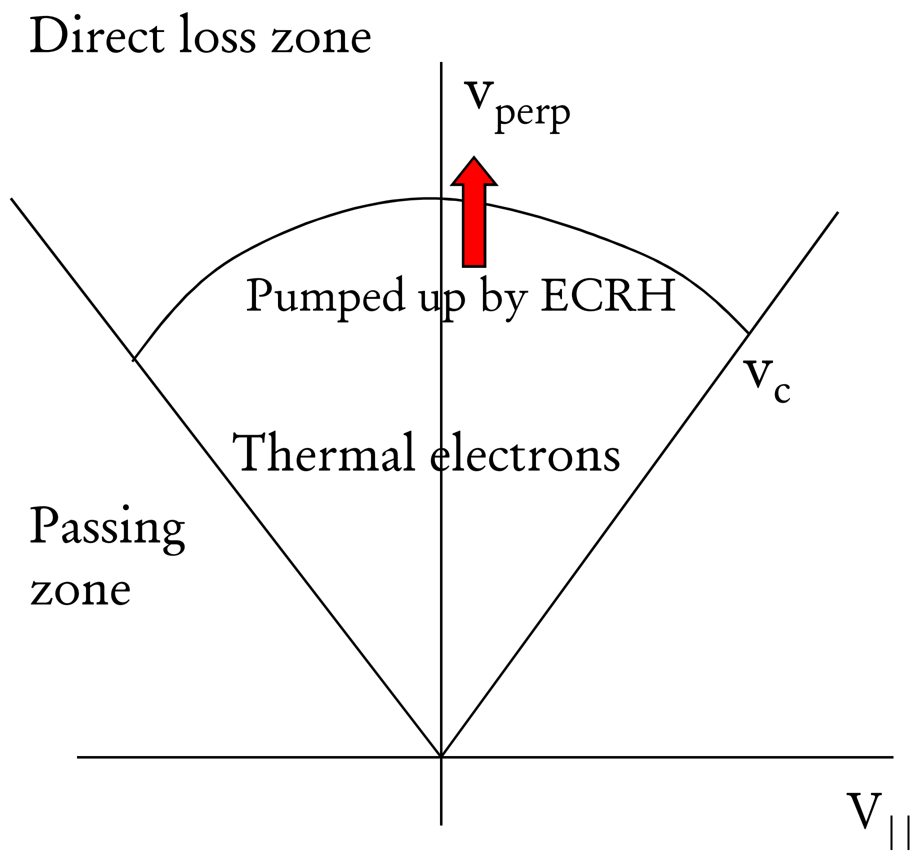

FIG. 3: Electrons above the critical energy are generated by ECRH. Trapped electrons are accelerated into the direct loss zone by ECRH. The minimum speed varies with the radial electric field E_r [Eq. (30) and Fig. 2].

Combining Eqs. (17), (19) and (21), we obtain

$$\left\langle \frac{\Delta v_\perp}{\Delta t} \right\rangle \approx \begin{cases} \frac{\sqrt{\pi} N}{8R} \frac{k v_\perp^{3/2} E_0}{B_0 \Omega'^{1/2}} \frac{1}{(\kappa^2 - \kappa_0^2)^{1/2} K(\kappa)} \exp\left[-\frac{1}{8L_b^2} \left(\frac{\Omega_0}{\Omega'}\right)^2 \frac{v^4}{c^4} \right], & \kappa > \kappa_0, \\ \frac{\sqrt{\pi} N}{2R} \frac{k^2 v_\perp^2 E_0}{B_0 \Omega'^{1/2}} \frac{L_b}{L} \frac{1}{K(\kappa_0)} \exp\left[-\frac{1}{8L_b^2} \left(\frac{\Omega_0}{\Omega'}\right)^2 \frac{v^4}{c^4} \right], & \kappa \approx \kappa_0, \\ 0, & \kappa < \kappa_0. \end{cases} \quad (35)$$

Since $\langle \Delta v_\perp / \Delta t \rangle$ is highly localized around κ_0, the pitch angle dependence part of $\langle \Delta v_\perp / \Delta t \rangle$ may be approximated using a delta function:

$$\left\langle \frac{\Delta v_\perp}{\Delta t} \right\rangle \approx \frac{\sqrt{\pi} N}{4R} \frac{k v_\perp^{3/2} E_0}{B_0 \Omega'^{1/2}} \frac{\delta(\kappa^2 - \kappa_0^2)}{K(\kappa)} \exp\left[-\frac{1}{8L_b^2} \left(\frac{\Omega_0}{\Omega'}\right)^2 \frac{v^4}{c^4} \right]. \quad (36)$$

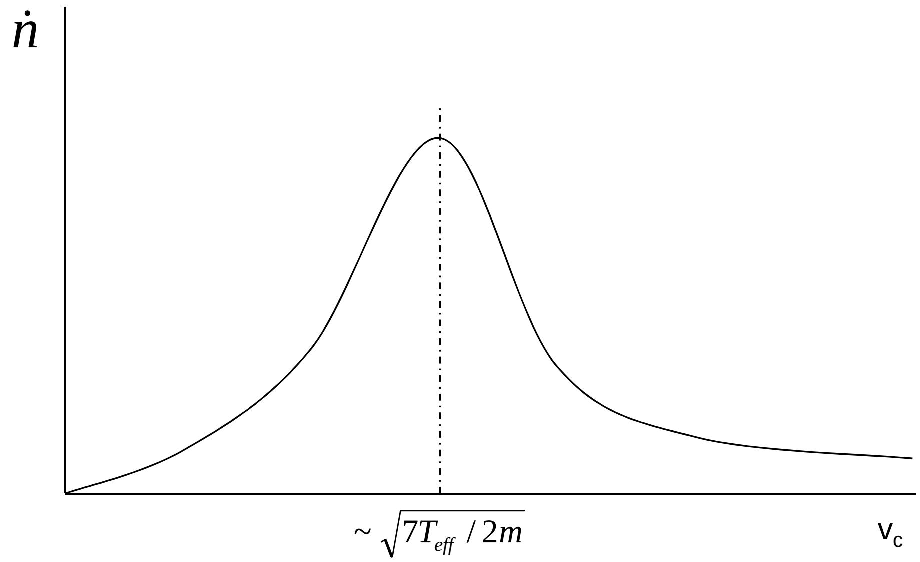

FIG. 4: The direct loss flux versus v_c. A simple model suggested by Fisch [15] is used to estimate the distribution function.

Using $d^2v = 2\pi v^2 \epsilon_H d\kappa^2$, Eq. (33) becomes

$$\dot{n} \approx -\frac{\pi^{3/2} M}{2R} \frac{k E_0 v_c^{7/2}}{B_0 \Omega'^{1/2}} \frac{\epsilon_H}{K(\kappa_0)} \exp\left[-\frac{1}{8L_b^2} \left(\frac{\Omega_0}{\Omega'}\right)^2 \frac{v_c^4}{c^4} \right] f(v_c, \kappa_0^2). \tag{37}$$

Neglecting the relativistic effect in the exponential, the coefficient of $\dot{n}$ is a strong function of v_c, the critical velocity for direct orbit losses. The population of trapped electrons increases as v increases if the distribution function is constant over the entire velocity space. Moreover, the quasi-linear diffusion coefficient is proportional to $v_\perp^2$. The direct loss flux is proportional to the electric field strength of the microwaves. The direct loss flux eventually decreases since it is dependent on the value of the distribution function at v_c. A simple model for rf heating previously suggested by Fisch [15] is used to estimate the distribution function. By adding quasi-linear diffusion to the Lenard-Bernstein model Fokker-Planck equation [16], we obtain

$$f(v, \kappa^2) = f(0, \kappa^2) \exp\left[-\int dv \frac{mv}{T_{eff}} \right], \tag{38}$$

where $T_{eff} = T + mD\nu$, using $D = D(\kappa)$ from Eq. (24). Figure 4 shows that when the Fisch model distribution function is used the direct loss flux peaks around $v_c \sim \sqrt{7T_{eff}/2m}$ and goes to zero as v_c increases.

IV. THE AMBIPOLAR ELECTRIC FIELD IN BUMPY STELLARATORS

The radial electric field is usually determined self-consistently from the ambipolarity condition on the particle fluxes. In steady state, the ambipolarity condition is given by

$$\sum_j Z_j \Gamma_j^{na} = 0, \tag{39}$$

where Γ_j^{na} is the non-ambipolar flux of particles of species j.

In the previous section, the direct loss flux driven by ECRH was estimated. The direct loss flux is another source of non-ambipolar particle flux. It is included in an extended ambipolarity condition to calculate the radial electric field. The density equation for species j particles is

$$\frac{\partial n_j}{\partial t} + \nabla \cdot \Gamma_j = -\dot{n}, \tag{40}$$

where $\dot{n}$ is the direct loss rate. Combining equilibrium equations for electrons and ions, we obtain

$$\nabla \cdot [\Gamma_e - Z_i \Gamma_i] = -\dot{n}. \tag{41}$$

Equation (41) can be integrated over the volume enclosed by a magnetic surface to yield

$$\Gamma_e - Z_i \Gamma_i \approx -\frac{\int dV \dot{n}}{4\pi^2 r R} \equiv \Gamma_{loss}, \tag{42}$$

where Γ_{loss} is the direct loss flux. The direct loss flux from a magnetic surface is obtained by integrating the direct loss over the volume enclosed by a magnetic surface and averaging over a magnetic surface. In Fig. 5, the geometry of cyclotron resonance is sketched. Electrons gain energy when they pass the intersection of a resonant surface and the microwave beam. We thus rewrite the integration of the direct loss in Eq. (42) as

$$\dot{N}(v_c, \kappa_0^2) = \int dV \dot{n}(\vec{r}, v_c, \kappa_0^2) \simeq 2 \int_{-\frac{\pi}{N}}^{\frac{\pi}{N}} R d\phi \int dx \int_{-L_b}^{L_b} dy \; \dot{n}(\vec{r}, v_c, \kappa_0^2). \tag{43}$$

When spatial inhomogeneity of the magnetic field strength in the x direction is considered, the gyrofrequency of electrons on the poloidal crosssection may be expressed as

$$\Omega(x) = \Omega_{res} + \Omega_0 x, \tag{44}$$

where $\Omega_0 = eB_0/m$. Equation (37) is modified to include the inhomogeneity of the magnetic field strength in x direction:

$$\dot{n}(\vec{r}, v_c, \kappa_0^2) \approx -\frac{\pi^{3/2} M}{2R} \frac{k E_0 v_c^{7/2}}{B_0 \Omega'^{1/2}} \frac{\epsilon_H}{K(\kappa_0)} \exp\left[-\frac{1}{8L_b^2} \left(\frac{\Omega_0}{\Omega'}\right)^2 \frac{v_c^4}{c^4} \right] f(v_c, \kappa_0^2) \sqrt{\pi} \frac{\Omega'}{\Omega_0} \delta(x - x_0). \tag{45}$$

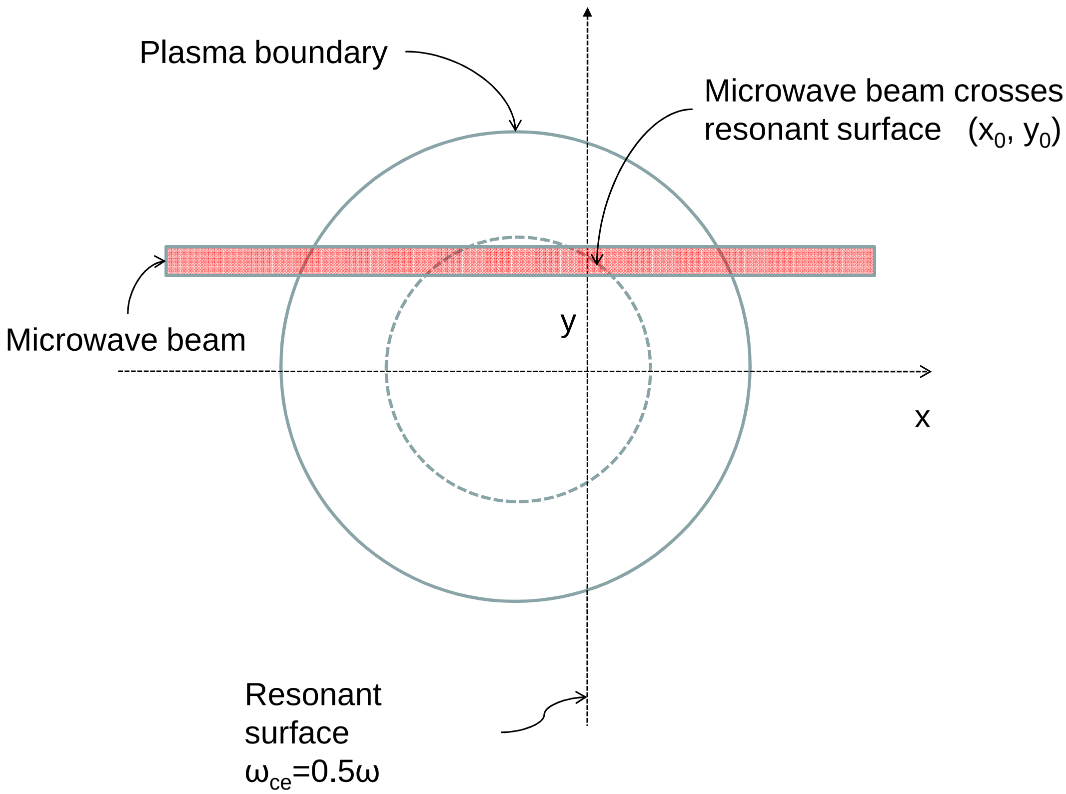

FIG. 5: Geometry for electron cyclotron resonance in the poloidal cross-section of a torus. Since the magnetic field inhomogeneity is in the x direction, the resonance occurs where the resonant surface intersects the microwave beam.

Then, the loss flux of electrons of a magnetic surface becomes

$$\Gamma_{loss}(r) \approx -\frac{1}{nr}\frac{\pi M}{2R}\frac{kE_0 v_c^{7/2}\Omega'^{1/2}}{B_0\Omega_0}\frac{\epsilon_H L_b}{K(\kappa_0)}\exp\left[-\frac{1}{8L_b^2}\left(\frac{\Omega_0}{\Omega'}\right)^2\frac{v_c^4}{c^4}\right]f(r_0, v_c, \kappa_0^2) \qquad (46)$$

when $r \geq r_0$. ECRH generates no loss flux inside the magnetic surface on which intersections of the resonant surface and the microwave beam lie. The loss flux is inversely proportional to the minor radius outside of the resonance zone.

The self-consistent radial electric field E_r is determined by Eq. (42), which is a modified ambipolarity condition that includes the ECRH-induced direct loss flux of electrons. Since the radial electric field reduces the direct loss via $v_c(E_r)$, the direct loss flux depends on the radial electric field. Electrons whose energy are high enough to drift out of the device without collisions can be confined by the poloidal rotation caused by the radial electric field. Therefore, the direct loss flux is constant with respect to the radial electric field strength E_r, until E_r is large enough to confine electrons with the temperature T_c. Figure 6 shows that the direct loss flux decreases as E_r increases when E_r is large enough to confine electrons

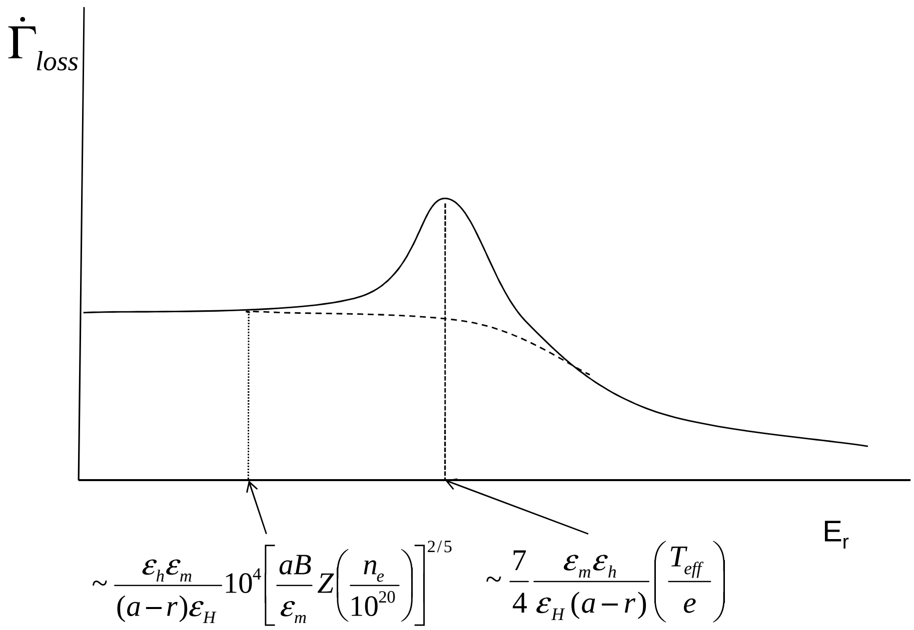

FIG. 6: The direct loss flux versus E_r. The solid line indicates the case with a hump in the direct loss flux. The dashed line indicates the case that the direct loss flux decreases without a hump.

with energies of about $\mathcal{E}_c$. A hump occurs when

$$T_{eff}/e > 0.57 \left[\frac{aB}{\epsilon_m} \nu_{ei}\right]^{2/5} T_e^{3/5} = 0.57 \cdot 10^4 aBZ(n_e/10^{20})^{2/5}/\epsilon_m. \tag{47}$$

When $T_{eff}/e < 0.57 \cdot 10^4 aBZ(n_e/10^{20})^{2/5}/\epsilon_m$, the direct loss flux decreases without a hump in it.

If all species in a stellarator are in the $1/\nu$ regime, the ambipolarity equation yields only one negative root. The electric field is negative because ions leave the device more quickly than electrons. Therefore, this is called the ion root. If all species are in the ν regime, the ambipolarity equation yields only one positive root. Since electrons tend to leave more quickly than ions in this regime, the electric field is positive and we call this the electron root. However, when the ions are in the ν regime and the background electrons are in the $1/\nu$ regime, the ambipolarity condition can have multiple solutions. The direct loss flux increases the total non-ambipolar flux of electrons. Thus, the direct loss flux would help the ambipolarity equation yield a large electron root, as indicated in Figure 7.

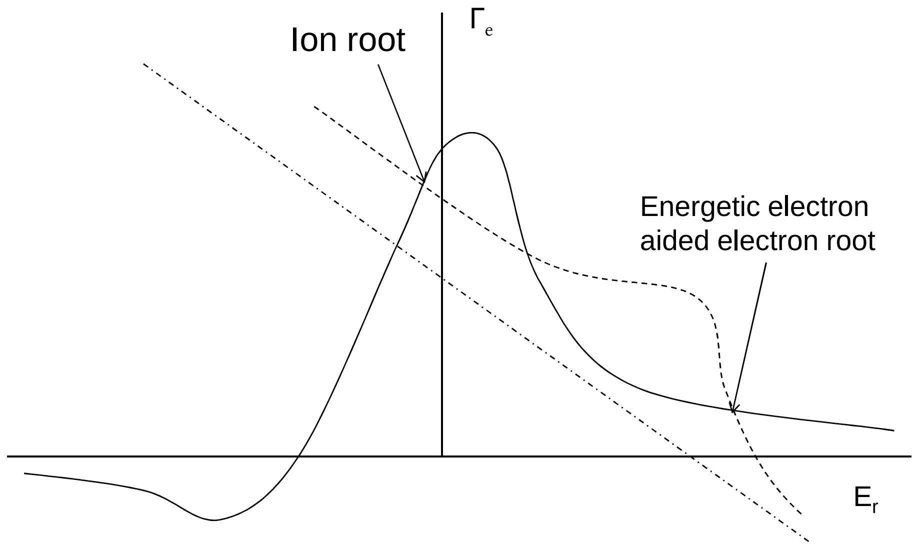

FIG. 7: The electron flux and the ion flux as functions of E_r. The solid line indicates the ion flux when they are in the ν regime. The dash-dotted line indicates the electron flux without ECRH when they are in the $1/\nu$ regime. The dotted line indicates the ECRH direct loss effects. Generally, the ambipolarity condition, Eq. (42), finds two stable roots for E_r.

V. CONCLUSIONS

The quasi-linear diffusion coefficient for second harmonic X-mode ECRH is localized in pitch angle. The maximum heating occurs when the turning points of trapped particles coincide with the resonance region. Nonlinear interaction occurs only when the microwave beam is focused on the bottom of the magnetic well and the pitch angles of trapped electrons are very close to 90 degrees. Therefore, the nonlinear interaction between waves and particles are usually negligible in second harmonic ECRH in bumpy stellarators.

A direct loss flux is generated by second harmonic X-mode ECRH in bumpy stellarators. The microwave field accelerates trapped electrons into the loss zone in velocity space. An ECRH-induced direct loss flux is an extra non-ambipolar flux in bumpy stellarators. The direct loss flux is proportional to the microwave field strength and the value of the distribution function at the critical speed v_c, above which direct losses occur. The direct loss flux increases the total non-ambipolar electron flux and ultimately can help the ambipolarity equation have multiple roots when background electrons are in the $1/\nu$ regime and ions are in the ν regime. Thus, a large positive radial electric field may be obtainable with the ECRH-induced direct loss flux.

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