UW-CPTC 07-5

Paleoclassical Electron Heat Transport Model

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(Dated: December 14, 2007)

Abstract

The paleoclassical model for radial electron heat transport in tokamak plasmas has been developed in a number of papers over the past few years. In order to facilitate numerical evaluation of it in plasma transport codes, this report summarizes its parameters, formulas and limits in particular regimes. A hierarchy of levels of the paleoclassical model are specified for interpretive (1) and predictive (2) transport codes: A) basic electron heat diffusivity $\chi_e^{\rm pc}$, B) effective power balance $\chi_{e{\rm PB}}^{\rm pc}$, C) internal transport barriers around low order rational surfaces, and D) different radial transport operator $\langle \boldsymbol{\nabla} \cdot \mathbf{q}_e^{\rm pc} \rangle$ that naturally embodies heat pinch effects. The approximations and limitations of the model are also discussed. Finally, possible future extensions of the paleoclassical model to other plasma transport processes, particularly density transport, and separatrix regions of divertor plasmas are discussed.

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I. INTRODUCTION

The paleoclassical model for radial electron heat transport has been posited [1] and developed using an axisymmetric toroidal magnetic field model [2]. Its main experimental predictions have been discussed [3] and many have been validated via transport modeling of experimental data [4, 5]. Also, a derivation of the key hypothesis of the paleoclassical model, which is that electron guiding centers diffuse along with thin annuli of poloidal magnetic flux, has been published [6, 7]. In view of the increasing maturity of the paleoclassical model, the time has come to provide a single coherent description of the model in a form that is useful for numerical evaluation in interpretive and predictive plasma transport codes.

It is important to note that the paleoclassical model, like the neoclassical model, provides a minimum level of plasma transport, particularly for radial electron heat transport. Anomalous transport induced by plasma microturbulence presumably just adds to it [2, 3]. Assuming the microturbulence-induced transport scales with the gyro-Bohm coefficient, paleoclassical electron heat transport is likely to be dominant for [4]

$$T_e \lesssim T_e^{\rm crit} \simeq B({\rm T})^{2/3}\, \bar{a}({\rm m})^{1/2}\ {\rm keV}, \quad \text{paleo electron heat transport dominant?} \qquad (1)$$

While $T_e^{\rm crit} \sim 0.7$–2.4 kev in present experiments, it is projected to be of order 3.5–5 keV in ITER. Thus, paleoclassical radial electron heat transport may be dominant in ohmic-level plasmas, ITER start-up plasmas, H-mode edge pedestals, and whenever microturbulence-induced anomalous transport is suppressed (e.g., in internal transport barrriers).

This report summarizes the formulas that represent paleoclassical radial electron heat transport. Section II describes the basic paleoclassical formulas and parameters. Section III derives the effective "power balance" paleoclassical electron heat diffusivity $\chi_{e\rm PB}^{\rm pc}$ to be compared with the $\chi_{e\rm PB}$ usually used in plasma transport codes. These two sections conclude with suggested orders for evaluating the various paleoclassical model parameters in transport codes. Section IV discusses the form and implications of the different transport operator embodied in the paleoclassical model, which naturally involves pinch-type effects. Sections V and VI discuss the special properties near low order rational surfaces, and the approximations and limitations involved in the paleoclassical model. The final regular section (VII) discusses possible future extensions of the paleoclassical model to other transport processes — density, ion heat and momentum transport, and transient processes. The summary (VIII) specifies a suggested hieracrchy of paleoclassical plasma transport models.

II. BASIC PALEOCLASSICAL FORMULAS AND PARAMETERS

The fundamental parameter of the paleoclassical transport model is [2, 3]

$$\boxed{D_\eta \equiv \frac{\eta_\parallel^{\rm nc}}{\mu_0} = \frac{\eta_0}{\mu_0}\,\frac{\eta_\parallel^{\rm nc}}{\eta_0},} \quad \text{magnetic field diffusivity, m}^2\text{/s.} \tag{2}$$

Here, $\eta_\parallel^{\rm nc}$ is the parallel neoclassical resistivity, $\mu_0 \equiv 4\pi \times 10^{-7}$ N/A^2 (in SI, MKS units) and $\eta_0 \equiv m_e \nu_e / n_e e^2$ is the reference, perpendicular resistivity, which can be written in the form of a magnetic field diffusivity as

$$\frac{\eta_0}{\mu_0} \simeq \frac{1400\,Z}{[T_e({\rm eV})]^{3/2}}\left(\frac{\ln\Lambda}{17}\right), \quad \text{reference magnetic field diffusivity, m}^2\text{/s.} \tag{3}$$

Here, $Z \to Z_{\rm eff} \equiv \sum_i n_i Z_i^2/n_e$ is the effective ion charge and $\ln\Lambda$ is the usual Coulomb logarithm. The parallel neoclassical resistivity $\eta_\parallel^{\rm nc}$ (in Ohm-m) can be evaluated using formulas in the literature [8, 9], or evaluated by the NCLASS code [10]. An approximate formula for it (relative to η_0) is [2, 3]

$$\frac{\eta_\parallel^{\rm nc}}{\eta_0} \simeq \frac{\eta_\parallel^{\rm Sp}}{\eta_0} + \frac{\mu_e}{\nu_e}, \quad \text{parallel neoclassical resistivity factor,} \tag{4}$$

in which the two components of the parallel resistivity in a tokamak are [2, 3]

$$\frac{\eta_\parallel^{\rm Sp}}{\eta_0} \simeq \frac{\sqrt{2}+Z}{\sqrt{2}+13Z/4}, \quad \text{Spitzer parallel resistivity factor, and} \tag{5}$$

$$\frac{\mu_e}{\nu_e} \simeq \frac{Z+\sqrt{2}-\ln(1+\sqrt{2})}{Z\,(1+\nu_{*e}^{1/2}+\nu_{*e})}\,\frac{f_t}{f_c} \overset{\nu_{*e}\to 0}{\underset{Z=1}{\Longrightarrow}} 1.5\,\frac{f_t}{f_c}, \quad \text{parallel electron viscosity effects.} \tag{6}$$

Here, f_c is the flow-weighted fraction of circulating particles [12] with Padé approximate [13]

$$f_c \simeq \frac{(1-\epsilon^2)^{-1/2}\,(1-\epsilon)^2}{1+1.46\epsilon^{1/2}+0.2\epsilon} \simeq 1 - 1.46\epsilon^{1/2} + \mathcal{O}(\epsilon), \quad \text{circulating particle fraction,} \tag{7}$$

in which $\epsilon \equiv (B_{\rm max} - B_{\rm min})/(B_{\rm max} + B_{\rm min}) \simeq r/R_0$ is the local inverse aspect ratio. Also, the fraction of trapped particles is $f_t \equiv 1 - f_c$ and

$$\nu_{*e} \equiv \frac{\nu_e}{\epsilon^{3/2}(v_{Te}/R_0 q)} = \frac{R_0 q}{\epsilon^{3/2}\lambda_e}, \quad \text{neoclassical electron collisionality parameter,} \tag{8}$$

in which the electron Coulomb collision "mean free path" is

$$\lambda_e \equiv \frac{v_{Te}}{\nu_e} \simeq 1.2 \times 10^{16}\,\frac{[T_e({\rm eV})]^2}{Z\,n_e({\rm m}^{-3})}\left(\frac{17}{\ln\Lambda}\right), \quad \text{electron collision length, m.} \tag{9}$$

The $\eta_\parallel^{\rm nc}$ from (4)–(9) ranges from being about equal (better than $1/\ln\Lambda \sim 6\%$ accuracy for $\mu_e/\nu_e << 1$) to as much as twice as large as (for $\mu_e/\nu_e >> 1$) the most precise neoclassical resistivity results [8, 9, 11, 12].

The flux-surface-averaged ($\langle\,\rangle$) paleoclassical radial electron heat transport operator is not in the usual form of the divergence of a radial electron heat flux. Rather, it is a multiplier times a divergence, which results from electron guiding centers and electron heat being carried radially along with diffusing annuli of poloidal magnetic flux (see Eq. (142) in [2]):

$$\boxed{\langle \boldsymbol{\nabla\cdot} \mathbf{q}_e^{\rm pc}\rangle \equiv \frac{\partial}{\partial V}\langle \mathbf{q}_e^{\rm pc}\boldsymbol{\cdot\nabla} V\rangle = \frac{1}{V'}\frac{\partial}{\partial\rho}\langle \mathbf{q}_e^{\rm pc}\boldsymbol{\cdot\nabla} V\rangle = -\,\frac{M+1}{V'}\,\frac{\partial^2}{\partial\rho^2}\left(V'\frac{D_\eta}{\bar{a}^2}\,\frac{3}{2}\,n_e T_e\right),} \quad \frac{\rm W}{\rm m^3}. \tag{10}$$

Here, $V' \equiv dV/d\rho$ (m^{-3}) is the radial derivative of the volume $V(\rho)$ (m^{-3}) of the flux surface whose dimensionless radial coordinate $\rho \equiv \sqrt{\psi_t/\psi_t(a)}$ is based on ψ_t, the toroidal magnetic flux per unit 2π, which at the limiter or divertor separatrix ($\rho = 1$) is defined by $\psi_t(a) \equiv \pi B_0 a^2/2\pi = B_0\, a^2/2$ in which B_0 is the (toroidal) magnetic field at the magnetic axis ($\rho = 0$). Also, $1/\bar{a}^2 \equiv \langle|\boldsymbol{\nabla}\rho|^2/R^2\rangle/\langle R^{-2}\rangle \simeq \langle\,|\boldsymbol{\nabla}\rho|^2\rangle[1+\mathcal{O}\{\epsilon^2\}] \simeq (1/a^2)(1+\kappa^2)/2\kappa^2$ in which $\kappa(\rho) \equiv b/a$ is the local (i.e., ρ-dependent) ellipticity of the plasma cross-section. Thus, it is convenient to define [2] a flux-surface-dependent minor radius

$$\boxed{\bar{a}(\rho) \equiv \left(\frac{1}{\langle|\boldsymbol{\nabla}\rho|^2/R^2\rangle/\langle R^{-2}\rangle}\right)^{1/2} \simeq \left(\frac{1}{\langle|\boldsymbol{\nabla}\rho|^2\rangle}\right)^{1/2} \simeq a\left(\frac{2\,\kappa^2}{1+\kappa^2}\right)^{1/2},} \quad \text{effective radius, m.} \tag{11}$$

The multiplier M in (10) is, apart from reductions near low order rational surfaces for time scales longer than the magnetic diffusion time scale $\tau_\eta \sim \bar{a}^2/6D_\eta$ (see Section V), given by the smoothed formula [2–4]

$$\boxed{M \simeq \frac{1/(\pi\bar{R}q)}{1/\ell_{\rm max}+1/\lambda_e} \sim \frac{\min\{\ell_{\rm max},\,\lambda_e\}}{\pi\bar{R}q},} \quad \text{helical multipler.} \tag{12}$$

The multiplier M is caused by helically resonant radial diffusion [2] in the vicinity of medium order ($q = m/n$, with $n \lesssim 10$) rational surfaces. It is the minimum of the collision length λ_e defined in (9) and an effective parallel length over which field lines are diffusing radially:

$$\ell_{\rm max} \equiv \pi\bar{R}q\,n_{\rm max}, \quad \text{parallel length of diffusing field lines, m,} \tag{13}$$

in which the maximum order n of the medium order rational surfaces is [2]

$$n_{\max} \equiv \left(\frac{1}{\pi \bar{\delta}_e |q'|} \right)^{1/2}, \quad \text{maximum } n \text{ for diffusing field lines.} \tag{14}$$

Here, $\bar{\delta}_e \equiv \delta_e / \bar{a}$ is a dimensionless electromagnetic (em) skin depth factor with

$$\boxed{\bar{\delta}_e \equiv \frac{\delta_e}{\bar{a}},} \quad \delta_e \equiv \frac{c}{\omega_p} \simeq 10^{-3} \left(\frac{3 \times 10^{19}}{n_e(\mathrm{m}^{-3})} \right)^{1/2} \mathrm{m} \sim 10^{-3} \ \mathrm{m}, \quad \text{em skin depth, m.} \tag{15}$$

Also, $q' \equiv \partial q(\rho, t)/\partial \rho$ in which q is the "safety factor" (toroidal winding number or inverse of field line pitch). In the vicinity of flux surfaces at which q' vanishes (at $\rho = 0$ and around off-axis minima, or extrema, in q), $n_{\max}$ is limited [2] (here, $q'' \equiv \partial^2 q / \partial \rho^2 |_{\rho_{\min}}$):

$$\max\{n_{\max}\} = \left(\frac{1}{\pi^2 \bar{\delta}_e^2 |q''|} \right)^{1/3}, \quad \text{maximum } n_{\max} \text{ near } \rho_{\min} \text{ where } q'(\rho_{\min}) = 0. \tag{16}$$

This maximum $n_{\max}$ is likely to be less than the $n_{\max}$ in (14) near where $q' = 0$ for

$$|\rho - \rho_{\min}| \lesssim \delta x_{\min} \equiv \left(\frac{\pi \bar{\delta}_e}{|q''|} \right)^{1/3}, \quad \text{radial region where } \max\{n_{\max}\} \lesssim n_{\max} \text{ in (14).} \tag{17}$$

The average major radius $\bar{R}$ is defined [2] by

$$\bar{R}(\rho) \equiv \frac{\langle B \rangle V'}{4\pi^2 \, \partial \psi_t / \partial \rho} \simeq R_0 [1 + \mathcal{O}\{\epsilon^2\}], \quad \text{effective major radius, m.} \tag{18}$$

Finally, the characteristic paleoclassical radial electron heat diffusivity is [2]

$$\boxed{\chi_e^{\mathrm{pc}} \equiv \frac{3}{2} (M+1) \, D_\eta = \frac{3}{2} (M+1) \frac{\eta_0}{\mu_0} \frac{\eta_\parallel^{\mathrm{nc}}}{\eta_0},} \quad \text{paleo electron heat diffusivity.} \tag{19}$$

In developing a computer routine to evaluate the characteristic paeoclassical electron heat diffusivity χ_e^{pc}, two passes through the profile data will be required. The first pass needs to identify the points where $q' \equiv \partial q / \partial \rho$ vanishes (at $\rho = 0$ and around off-axis extrema in q), and then determine the $\max\{n_{\max}\}$ from (16) and the likely approximate radial extent $\delta x_{\min}$ from (17) at those points. Then, on the second pass the suggested order of evaluation of the various quantities (D_η, $\bar{\delta}_e$, $n_{\max}$, $\ell_{\max}$, λ_e, and M) to determine χ_e^{pc} at each ρ is:

1) Evaluate $D_\eta \equiv \eta_\parallel^{\mathrm{nc}} / \mu_0$ using $\eta_\parallel^{\mathrm{nc}}$ from (3)–(9), or (preferably) from NCLASS [10] or [8, 9];

2) Use the approximate forms of $\bar{a}$ and $\bar{R}$ from (11) and (18), or evaluate them from their respective definitions in those equations (save $1/\bar{a}^2$ for evaluations in the next section);

3) Evaluate the electromagnetic skin depth δ_e and calculate $\bar{\delta}_e \equiv \delta_e / \bar{a}$ from (15);

4) Calculate $n_{\rm max}$ from (14), but bound it by the value given by (16) in the vicinity of flux surfaces (ρ values) where $q' \equiv \partial q/\partial\rho = 0$ (at $\rho = 0$ and around off-axis extrema in q);
5) Determine the length $\ell_{\rm max}$ from (13) and the electron collision length λ_e from (9);
6) Obtain a value for the helical multiplier M using the first form in (12); and
7) Finally, evaluate $\chi_e^{\rm pc}$ from (19).

III. EFFECTIVE POWER BALANCE DIFFUSIVITY

The paleoclassical radial electron heat transport operator in (10) is not in the usual form of the divergence of a Fourier (superscript F) heat flux $\mathbf{q}_e^{\rm F} \equiv -n_e\chi_e\boldsymbol{\nabla}T_e$. However, the Fourier heat flux form is usually used to determine a "power balance" $\chi_{e\rm PB}$ from the electron heat flow $P_e(\rho)$ through a flux surface divided by $-n_e|\boldsymbol{\nabla}T_e|$. Thus, an effective $\chi_{e\rm PB}^{\rm pc}$ is needed for direct comparisons to the usual experimentally-inferred $\chi_{e\rm PB}$ obtained from interpretive transport analysis. Using $\boldsymbol{\nabla}V = V'\boldsymbol{\nabla}\rho$ and $\boldsymbol{\nabla}T_e = (\partial T_e/\partial\rho)\boldsymbol{\nabla}\rho$ to yield $\langle\mathbf{q}_e^{\rm F}\cdot\boldsymbol{\nabla}V\rangle \equiv -n_e\chi_{e\rm PB}\langle\boldsymbol{\nabla}V\cdot\boldsymbol{\nabla}T_e\rangle = -n_e\chi_{e\rm PB}V'\langle|\boldsymbol{\nabla}\rho|^2\rangle\partial T_e/\partial\rho$, one usually defines

$$\chi_{e\rm PB} \equiv \frac{\langle\mathbf{q}_e\cdot\boldsymbol{\nabla}V\rangle_\rho}{\langle -n_e\boldsymbol{\nabla}T_e\cdot\boldsymbol{\nabla}V\rangle} = \frac{P_e(\rho)}{n_eV'\langle|\boldsymbol{\nabla}\rho|^2\rangle\,(-\,\partial T_e/\partial\rho)}, \quad \text{power balance (PB) } \chi_e,\ \text{m}^2/\text{s}. \tag{20}$$

Integrating (10) over volume ($d^3x \equiv dV \equiv V'd\rho$) from the magnetic axis, where $\rho = 0$ and the radial heat flow vanishes ($\langle\mathbf{q}_e^{\rm pc}\cdot\boldsymbol{\nabla}V\rangle \sim \rho^2$ as $\rho \to 0$ because $\langle\boldsymbol{\nabla}\cdot\mathbf{q}_e^{\rm pc}\rangle \sim$ constant as $\rho \to 0$), to ρ yields the paleoclassical radial electron heat flow through the ρ flux surface:

$$P_e^{\rm pc}(\rho) \equiv \langle\mathbf{q}_e^{\rm pc}\cdot\boldsymbol{\nabla}V\rangle_\rho = -\int_0^\rho d\rho\,(M+1)\,\frac{\partial^2}{\partial\rho^2}\left(V'\frac{D_\eta}{\bar{a}^2}\frac{3}{2}n_eT_e\right), \quad \text{core paleo heat flow, W.} \tag{21}$$

Alternatively, integrating from ρ to the divertor magnetic separatrix ($\rho = 1$), one obtains

$$P_e^{\rm pc}(\rho) = \langle\mathbf{q}_e^{\rm pc}\cdot\boldsymbol{\nabla}V\rangle_1 + \int_\rho^1 d\rho\,(M+1)\,\frac{\partial^2}{\partial\rho^2}\left(V'\frac{D_\eta}{\bar{a}^2}\frac{3}{2}n_eT_e\right), \quad \text{edge paleo heat flow, W.} \tag{22}$$

These alternative forms represent the paleoclassical radial electron heat flow (in Watts) through the ρ flux surface. As one approaches a divertor separatrix, $q \to \infty$, so $M \to 0$ and $\langle\mathbf{q}_e^{\rm pc}\cdot\boldsymbol{\nabla}V\rangle_1 = -(\partial/\partial\rho)[V'(D_\eta/\bar{a}^2)(3/2)n_eT_e]_1$. Since $P_e^{\rm pc}(0)$ must vanish, one also has from (22) $\langle\mathbf{q}_e^{\rm pc}\cdot\boldsymbol{\nabla}V\rangle_1 = -\int_0^1 d\rho\,(M+1)(\partial^2/\partial\rho^2)[V'(D_\eta/\bar{a}^2)(3/2)n_eT_e]$; hence (21) and (22) are analytically equivalent. However, numerical evaluations of them near the edge could produce different results; in particular, if the n_e and T_e profiles near the separatrix are not known sufficiently precisely, the evaluation in (22) might be less accurate than (21).

For the paleoclassical model an effective $\chi_{e\mathrm{PB}}^{\mathrm{pc}}$ is obtained by dividing P_e^{pc} in (21) or (22) by $-n_e V' \langle |\boldsymbol{\nabla}\rho|^2\rangle \, \partial T_e/\partial\rho$. Using $P_e^{\mathrm{pc}}(\rho)$ from (21) in (20), one obtains [correct to $\mathcal{O}\{\epsilon^2\}$ where from (11) $1/\bar{a}^2 \simeq |\boldsymbol{\nabla}\rho|^2$]

$$\boxed{\chi_{e\mathrm{PB}}^{\mathrm{pc}} \equiv \frac{P_e^{\mathrm{pc}}(\rho)}{n_e V' \langle |\boldsymbol{\nabla}\rho|^2\rangle \, (-\,\partial T_e/\partial\rho)}}$$

$$\simeq \frac{-\int_0^\rho d\rho \, (M+1) \, \frac{\partial^2}{\partial\rho^2} \left(V' \frac{D_\eta}{\bar{a}^2} \frac{3}{2} n_e T_e \right)}{n_e V' (1/\bar{a}^2)(-\,\partial T_e/\partial\rho)}, \quad \text{paleo power balance } \chi_e, \ \mathrm{m}^2/\mathrm{s}. \qquad (23)$$

While $\chi_{e\mathrm{PB}}^{\mathrm{pc}}$ is obviously of order $\chi_e^{\mathrm{pc}} \equiv (3/2)(M+1)D_\eta$ in (19), it is not equal to it. In the hot core region (I) where one is usually in the collisionless ($\lambda_e > \ell_{\max}$) paleoclassical regime, $M \simeq n_{\max}$ is nearly constant and the $M+1$ factor can be taken out of the integral in (21) which can then be integrated. The result is that in the hot core region (I [4]) it is likely that $\chi_{e\mathrm{PB}}^{\mathrm{pc}} \lesssim \chi_e^{\mathrm{pc}}$ with a small heat pinch effect [3]. However, in the collisional ($\pi \bar{R} q \, n_{\max} > \lambda_e > \pi \bar{R} q$, Alcator-scaling) paleoclassical regime (region II), $M \simeq \lambda_e/\pi\bar{R}q$ decreases rapidly as ρ increases, P_e^{pc} can decrease with increasing ρ (cf., Fig. 6 in [4]) and $\chi_{e\mathrm{PB}}^{\mathrm{pc}}$ is likely less than χ_e^{pc} — by a factor of 2 or more? Finally, near a magnetic separatrix (region III) it is difficult to estimate $\chi_{e\mathrm{PB}}^{\mathrm{pc}}$, particularly when using (22) in (23) — because precise profiles of n_e and T_e there are required; but it is probably also less than χ_e^{pc} there.

Thus, while the paleoclassical radial electron heat transport operator can be forced into the usual radial heat flow and power balance χ_e formats, doing so produces forms different from the usual ones. The ratio $\chi_{e\mathrm{PB}}^{\mathrm{pc}}/\chi_e^{\mathrm{pc}}$ is likely to be about unity in the hot core (region I), but less than unity in regions II (Alcator-scaling regime, about top half of an H-mode pedestal) and maybe III (near separatrix) — perhaps by factors of 2 or more.

In developing a computer routine to evaluate the paleoclassical power balance $\chi_{e\mathrm{PB}}^{\mathrm{pc}}$, it is recommended that the power flow $P_e^{\mathrm{pc}}(\rho)$ be calculated first using (21) and compared to the $P_e(\rho)$ inferred from interpretive transport analysis of experimental data. Then, $\chi_{e\mathrm{PB}}^{\mathrm{pc}}$ can be readily obtained from the first form in (23). Note that $P_e^{\mathrm{pc}}(\rho)$ and $\chi_{e\mathrm{PB}}^{\mathrm{pc}}(\rho)$ could be determined at the end of the second pass through the profile data discussed at the end of the preceding paragraph after D_η, $1/\bar{a}^2$ and M have been computed in steps 1), 2) and 6) of that procedure. The only new data needed are $V'(\rho)$ and ρ derivatives of T_e and $V'(D_\eta/\bar{a}^2)n_e T_e$.

IV. PALEOCLASSICAL TRANSPORT OPERATOR

The usual electron heat transport operator is in the form of the divergence of a heat flux. For example, for a Fourier heat flux law $\mathbf{q}_e^{\mathrm{F}} = -n_e\chi_e\boldsymbol{\nabla}T_e$, one has (with $\rho \to r/\bar{a}$, $V' \propto r$)

$$\langle\boldsymbol{\nabla}\cdot\mathbf{q}_e^{\mathrm{F}}\rangle = \frac{\partial}{\partial V}\langle\mathbf{q}_e^{\mathrm{F}}\cdot\boldsymbol{\nabla}V\rangle = -\frac{1}{V'}\frac{\partial}{\partial\rho}\left(V'\langle|\boldsymbol{\nabla}\rho|^2\rangle n_e\chi_e\frac{\partial T_e}{\partial\rho}\right) \sim -\frac{1}{r}\frac{\partial}{\partial r}\left(r n_e\chi_e\frac{\partial T_e}{\partial r}\right). \tag{24}$$

However, the paleoclassical transport operator in (10) is not in this form. The extra first derivative term in (10) produces a heat pinch type effect [2–4, 6]. Thus, if one uses the effective $\chi_{e\mathrm{PB}}^{\mathrm{pc}}$ in (23) for χ_e in (24) and the usual predictive transport code solution procedures, one can anticipate there could be numerical iteration and convergence problems, particularly towards the plasma edge (regions II and III) where the helical multiplier M decreases substantially and eventually becomes negligible compared to unity near a separatrix.

To fully implement a paleoclassical electron heat transport model in a predictive transport simulation one should, in principle, change the numerical iteration scheme to take account of the form of the paleoclassical operator in (10) — in combination with the usual form in (24) representing neoclassical and anomalous transport processes. However, this could involve major revisions in the transport simulation code algorithms and be a daunting undertaking.

V. SPECIAL CONSIDERATIONS NEAR LOW ORDER RATIONAL SURFACES

In the general paleoclassical model [2], in the vicinity of low order rational surfaces where $q^\circ(\rho^\circ) = m^\circ/n^\circ$ with $n^\circ = 1, 2$, the length of a rational field line is only $\ell^\circ = \pi\bar{R}q\,n^\circ << \ell_{\max}$, and this often becomes the smallest length determining the factor M (for $\ell^\circ < \lambda_e$ [2, 3]). Then, one effectively has $n_{\max} \to n^\circ$ and hence $M \sim n^\circ$. (Since this result is obtained using an asymptotic analysis [2] that assumes $n >> 1$, M is only of order n°; an appropriate numerical factor is unknown.) The factor M remains small ($\sim n^\circ = 1, 2$), which indicates an electron internal transport barrier (eITB) around q°, up to a distance from the rational surface of $\bar{\delta}_e$ less than the distance $\delta x^\circ \equiv |\rho_{n_{\max}} - \rho^\circ|$ to the closest $n_{\max}$ rational surface $q(\rho_{n_{\max}}) \equiv m_{\max}/n_{\max}$ at $\rho_{n_{\max}}$. Neglecting $\bar{\delta}_e$, this distance is approximately [2, 3]

$$\delta x^\circ \simeq \frac{1}{n^\circ}\left(\frac{\pi\bar{\delta}_e}{|q'|}\right)^{1/2}, \quad \text{half-width of electron internal transport barrier (eITB)}, \tag{25}$$

or, around a minimum (or maximum) in q about $q^\circ \equiv m^\circ/n^\circ = q_{\rm min}$ [2, 3],

$$\delta x^\circ_{\rm min} \simeq \left(\frac{2}{n^\circ}\right)^{2/3} \left(\frac{\pi\bar{\delta}_e}{|q''|}\right)^{1/3}, \quad \text{half-width of eITB at } q_{\rm min} = m^\circ/n^\circ. \tag{26}$$

In most toroidal plasmas these paleoclassical electron internal transport barriers (eITBs) are not likely to be very important for four reasons: 1) They are likely to be rather narrow, unless there is an off-axis minimum in q. 2) Since the electron heating power is usually broadly distributed, the local changes in the electron temperature gradient induced by the eITBs are typically rather small. 3) Because the eITB characteristics depend sensitively on the magnitude and derivatives of the q profile, one needs the $q(\rho)$ profile to be very precisely determined and stationary in time, which requires equilibration on time scales longer than the global magnetic diffusion time scale $\tau_\eta \sim \bar{a}^2/6D_\eta(\rho=0)$. And 4) The widest possible eITBs would occur in reversed shear plasmas with an off-axis minimum in the q profile which are usually accessed via significant auxiliary heating that typically causes $T_e > T_e^{\rm crit}$.

These four points are well-illustrated by the only modeling of these low order rational surface paleoclassical effects to date — by Hogeweij [4, 5] for the RTP data on the response of $T_e(0)$ to highly localized ECH. Specifically, in the RTP modeling [4, 5], 1) for monotonic q profiles the widths $(2\,\delta x^\circ)$ of eITBs were narrow (less than 4% of the radius, except around $q^\circ = 1/1,\ 2/1,\ 3/1$ which could be about 7%); 2) the ECH was localized to 10% (FWHM) of the plasma radius; 3) the plasma required (both in the experiment and in the simulations) $\sim$ 20 ms ($\sim \tau_\eta$) to bifurcate to two different transport equilibria for the two very close ECH deposition radii of $\rho_{\rm dep} = 0.446$ and 0.447; and 4) the RTP plasmas had $T_e < T_e^{\rm crit} \sim 0.7$ keV over all but the hot center ($\rho \lesssim 0.25$) of the plasma. While the observations in RTP and modeling of eITBs around low order rational surfaces is a singular and remarkable demonstration of the veracity of paleoclassical electron heat transport, these eITBs are not likely to be very important in most tokamak plasmas — except when $T_e < T_e^{\rm crit}$ (or microturbulence-induced anomalous transport is suppressed) around a minimum $q_{\rm min} = m^\circ/1$ at a low order rational surface ($n^\circ = 1$) of a nearly stationary q profile.

VI. APPROXIMATIONS AND LIMITATIONS IN MODEL

Because the paleoclassical transport formulas were obtained using a large n asymptotic analysis and the characteristic lengths in M were only approximately determined, the formula for M, and hence all M-dependent results herein, should be interpreted as scaling results [2, 3]; future, more detailed studies could introduce "headache factor" numerical coefficients of order unity in M, $\chi_e^{\rm pc}$ and $\chi_{e\rm PB}^{\rm pc}$. Also, terms of order the inverse aspect ratio squared, i.e., $\mathcal{O}\{\epsilon^2\}$, and the square of the ratio of the poloidal to toroidal magnetic field, i.e., $\mathcal{O}\{B_p^2/B_t^2\} \sim \mathcal{O}\{\epsilon^2/q^2\}$, have been neglected [2]. Finally, the paleoclassical formulas have been determined for "near equilibrium" conditions and hence are only applicable [2] for a slowly changing (in space and time, i.e., for $t > \tau_\eta$) poloidal flux $\psi(\rho, t)$. Paleoclassical transport properties and formulas for transient situations have not yet been worked out; the changes are likely to be significant in situations where the q profile changes significantly — because of rapid changes in the parallel neoclassical resistivity $\eta_\parallel^{\rm nc}$ (and hence D_η) or in the current sources [2, 6] (rate of change of ohmic transformer flux, bootstrap current or non-inductive current drive). In view of all these caveats, the paleoclassical formulas and transport model should be considered to have "theoretical error bars" of about a factor of two [2, 4].

VII. POSSIBLE EXTENSIONS OF PALEOCLASSICAL MODEL

The original [1–3] and recent [6] research on the paleoclassical transport model have focused on establishing the basics of paleoclassical radial electron heat transport — because electron heat transport is the dominant paleoclassical process in tokamak plasmas and it can be dominant for $T_e \lesssim T_e^{\rm crit}$ [4, 5]. Extensions of the present paleoclassical model that are needed to make it more complete and useful for modeling the full range of plasma transport on closed magnetic flux surfaces inside a divertor separatrix are: 1) Development of paleoclassical density, momentum, and ion heat transport equations, including their natural pinch effects — to be able to model these other paleoclassical transport processes; 2) Determination of the non-ambipolar helically-resonant component of paleoclassical transport and the radial electric field E_ρ it implies [6] in the collisional (Alcator-scaling) regime (II) — to explore their possible role in "intrinsic" plasma toroidal rotation, particularly in H-mode pedestals; 3) More precise determination of the coefficients in and the form of the "smoothing formula" M in (12) — for greater precision in the paleoclassical predictions; 4) Consideration of a possible $\lambda_e \lesssim \pi R_0$ regime near a magnetic separatrix — to be able to model very collisional C-Mod H-mode pedestals [14]; 5) Development of a model for moving rational surfaces and other transient effects (see discussion in last paragraph of Section VII in [2]) — to be able to model transient transport experiments; 6) Inclusion of poloidal electron heat flows and ion flows — to obtain more accurate neoclassical resistivity and bootstrap current formulas and their effects in the paleoclassical model.

The other paleoclassical transport operators are apparently [6] similar to that in (10) except that there is no M factor, which is unique (see last of Section VI.C. in [2]) to electron heat transport in the paleoclassical model. Thus, for example, the (ambipolar [6]) paleoclassical density transport operator obtained from the density moment of the magnetic-diffusion-modified drift-kinetic equation (MDKE) derived in paleoclassical theory [2, 6] is

$$-\langle \boldsymbol{\nabla \cdot \Gamma}^{\rm pc} \rangle = \frac{1}{V'} \frac{\partial^2}{\partial \rho^2} \left(V' \frac{D_\eta}{\bar{a}^2} n_e \right) = \frac{1}{V'} \frac{\partial}{\partial \rho} \left[V' \left(\frac{D_\eta}{\bar{a}^2} \frac{\partial n_e}{\partial \rho} + n_e V_p^{\rm pc} \right) \right], \tag{27}$$

in which the intrinsic particle "pinch velocity" in the paleoclassical model is

$$V_p^{\rm pc} \equiv \frac{1}{V'} \frac{\partial}{\partial \rho} \left(V' \frac{D_\eta}{\bar{a}^2} \right), \quad \text{paleoclassical radial particle pinch velocity, s}^{-1}. \tag{28}$$

Since $D_\eta \propto 1/T_e^{3/2}$ typically increases strongly with radius, this pinch velocity usually: 1) is positive (inward), 2) increases with radius, and 3) is largest at the plasma edge. The effective "power balance" paleoclassical particle diffusivity is thus given by

$$D_{\rm eff}^{\rm pc} = D_\eta - \frac{\bar{a}^2 V_p^{\rm pc}}{-\partial \ln n_e / \partial \rho} = D_\eta \left(1 - \frac{(\partial/\partial\rho) \ln (V' D_\eta / \bar{a}^2)}{-(\partial/\partial\rho) \ln n_e} \right), \tag{29}$$

Similar transport operators with both diffusive and pinch-type fluxes will apparently also be obtained for the ion temperature T_i and plasma toroidal rotation rate $\Omega_\zeta \equiv \mathbf{V} \boldsymbol{\cdot} \boldsymbol{\nabla} \zeta \simeq V_\zeta / R$.

It is well known that a two-dimensional (2D, radial and "poloidal") model is needed to describe plasma transport in the vicinity (just inside, "on," and outside) of a divertor separatrix. Extensions of the present paleoclassical model that are needed to make it more complete and useful for modeling tokamak plasmas in the vicinity of a separatrix are: 1) Development of a paleoclassical formalism based on local field line (Clebsch-like) coordinates rather than the usual "global" magnetic flux coordinates ψ, θ, ζ — to facilitate calculating the "poloidal" variation of paleoclassical transport processes; 2) Procedures for extracting from magnetic equilibrium fitting codes such as EFIT the "current-driven" (ψ_J) and "vacuum" (ψ_V) components of the poloidal magnetic flux in the vicinity of a magnetic separatrix, which causes $D_\eta \to [(\partial \psi_J/\partial\rho)/(\partial\psi_J/\partial\rho + \partial\psi_V/\partial\rho)] D_\eta$ — because paleoclassical transport is only induced by the current-driven component of the poloidal flux ψ [6]; 3) Development of a useful "poloidally global" magnetic field coordinate system in the vicinity of a divertor magnetic separatrix, perhaps based on the type of analytic analysis used for magnetic islands [15] involving (incomplete) elliptic functions — to have a representation of the magnetic field that can be used uniformly from inside to outside the separatrix; 4) Determination of the "poloidal" variation of the "radial" paleoclassical electron heat transport — to facilitate exploring the 2D nature of both perpendicular and parallel [16] electron heat transport in the vicinity of the separatrix; 5) Determination of the corresponding "poloidal" dependence of the density, ion heat, momentum transport and flows — to facilitate exploring their 2D properties in the vicinity of the separatrix.

VIII. SUMMARY

The purpose of this report has been to specify the key formulas of the paleoclassical radial electron heat transport model — for evaluation and utilization of them in interpretive and predictive transport codes. Approximations involved in and possible extensions of the paleoclassical transport model have also been discussed in Sections VI and VII. A hierarchy of levels of evaluation and implementation of the paleoclassical model can be specified:

1) Paleoclassical radial electron heat transport parameters for interpretive codes:

A) Evaluate $\chi_e^{\rm pc}$ from (19), via the steps described at the end of Section II.

B) Evaluate $P_e^{\rm pc}$ and $\chi_{e{\rm PB}}^{\rm pc}$ from (21) and (23), as described at the end of Section III.

C) Include eITB effects on M around low order rational surfaces, as described in Section V.

2) Implementation of paleoclassical electron heat transport model in predictive codes:

A) Use $\chi_e^{\rm pc}$ from 1A above to represent the paleoclassical contribution to χ_e.

B) Use $P_e^{\rm pc}$ and $\chi_{e{\rm PB}}^{\rm pc}$ from 1B above to represent paleoclassical contributions to P_e and χ_e.

C) Include eITB effects on M around low order rational surfaces, as evaluated in 1C above.

D) Use the full paleoclassical electron heat transport operator in (10) — see Section IV.

3) Complete paleoclassical transport model (in future):

A) Add density, T_i, momentum transport effects — see discussions in Section VII and [6].

B) Add transient effects — see discussions in Section VII and [2].

C) Add separatrix region 2D effects — see discussion at end of Section VII.

ACKNOWLEDGEMENT

The author is very grateful to his many experimental and modeling colleagues [4] who developed early versions of computer routines for evaluating and testing paleoclassical radial electron heat transport. These colleagues pointed out many pitfalls in the initially suggested forms of the paleoclassical model and thereby helped develop many of the procedures suggested in this report. This research was supported by DoE grant DE-FG02-92ER54139.

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