Analysis of pedestal plasma transport

J D Callen,1 R J Groebner,2 T H Osborne,2 J M Canik,3 L W Owen,3 A Y Pankin,4 T Rafiq,4 T D Rognlien5 and W M Stacey6

1University of Wisconsin, Madison, WI 53706-1609 USA
2General Atomics, San Diego, CA 92186-5608 USA
3Oak Ridge National Laboratory, Oak Ridge, TN 37831 USA
4Lehigh University, Bethlehem, PA 18015-3182 USA
5Lawrence Livermore National Laboratory, Livermore, CA 94551-0808 USA
6Georgia Tech, Atlanta, GA 30332 USA

E-mail: callen@engr.wisc.edu, groebner@fusion.gat.com

Abstract. An H-mode Edge Pedestal (HEP) plasma transport Benchmarking Exercise (BE) was undertaken for a single DIII-D pedestal. Transport modeling codes used include 1.5D interpretive (ONETWO, GTEDGE), 1.5D predictive (ASTRA) and 2D ones (SOLPS, UEDGE). The particular DIII-D discharge considered is 98889, which has a typical low density pedestal. Profiles for the edge plasma are obtained from Thomson and CER data averaged over the last 20 % of the average 33.53 ms repetition time between Type I ELMs. The modeled density of recycled neutrals is largest in the divertor X-point region and causes the edge plasma source rate to vary by a factor $\sim 10^2$ on the separatrix. Modeled poloidal variations in the density and temperatures on flux surfaces are small on all flux surfaces up to within about 2.6 mm ($\rho_N > 0.99$) of the mid-plane separatrix. For the assumed Fick’s-diffusion-type laws, the radial heat and density fluxes vary poloidally by factors of 2–3 in the pedestal region; they are largest on the outboard mid-plane where flux surfaces are compressed and local radial gradients are largest. Convective heat flows are found to be small fractions of the electron (\lesssim 10 %) and ion (\lesssim 25 %) heat flows in this pedestal. Appropriately averaging the transport fluxes yields interpretive 1.5D effective diffusivities that are smallest near the mid-point of the pedestal. Their “transport barrier” minima are about 0.3 (electron heat), 0.15 (ion heat) and 0.035 (density) m$^2$/s. Electron heat transport is found to be best characterized by ETG-induced transport at the pedestal top and paleoclassical transport throughout the pedestal. The effective ion heat diffusivity in the pedestal has a different profile from the neoclassical prediction and may be smaller than it. The very small effective density diffusivity may be the result of an inward pinch flow nearly balancing a diffusive outward radial density flux. The inward ion pinch velocity and density diffusion coefficient are determined by a new interpretive analysis technique that uses information from the force balance (momentum conservation) equations; the paleoclassical transport model provides a plausible explanation of these new results. Finally, the measurements and additional modeling needed to facilitate better pedestal plasma transport modeling are discussed.

PACS numbers: 52.55.Fa, 52.25.Fi, 52.55.Dy, 52.55.Rk

Submitted to: Nuclear Fusion
1. Introduction

Large edge pedestals in high- (H-) confinement mode plasmas are critical [1] for achieving high fusion power in ITER [2]. At present, plasma transport processes and their role in H-mode pedestals are not well understood. In this work we seek to identify, clarify and quantify the key transport processes involved in H-mode pedestals in the quasi-equilibrium transport state between Type I ELMs. Specifically, an H-mode Edge Pedestal (HEP) Benchmarking Exercise (BE) for a single DIII-D pedestal has been undertaken in which transport modeling results from various types of modeling codes are extensively compared and benchmarked for a single, well-characterized, typical H-mode pedestal in the DIII-D tokamak [3]. The initial objectives of the HEP BE were to: 1) Determine channels of energy flow through and losses from the pedestal; 2) identify key physical processes for characterizing these energy flows to a) determine what key experimental measurements are most needed, b) clarify what processes need to be included in transport modeling, and c) increase confidence in transport modeling as various code results converge; and 3) facilitate meaningful, reliable comparisons with theory-based transport models.

Transport analyses are most straightforward when plasmas are in transport equilibrium. But H-mode pedestals evolve [4] between Edge Localized Modes (ELMs). There is a wide variability in the time scales for edge pedestals to re-build between ELMs. However, in an exploration [4] of pedestals after Type I ELMs in DIII-D that were slowly evolving, it was shown that after an ELM the average pedestal pressure gradient first increased “rapidly” on a time scale of about 10–20 ms to a “transport quasi-equilibrium;” it then increased more slowly for ∼ 30–100 ms, until the next ELM. An ∼ 10 ms transient phase is consistent with the time scale for diffusive transport over a thin pedestal width Δx ∼ 0.03 m for D ∼ 0.1 m^2/s: τ ∼ (Δx)^2/D ∼ 9 ms.

In this study we analyze plasma transport properties in the transport quasi-equilibrium phase just before the next ELM. The particular pedestal considered is from a typical low pedestal density DIII-D discharge. It will be characterized by data obtained by averaging over the last 20 % of the average 33.53 ms period between Type I ELMs from about 4 to 5 s in DIII-D discharge 98889. Plasma transport properties will be analyzed mainly from the core region inside the top of the pedestal (from normalized radius ρ_N ≃ 0.85) through the pedestal (0.94 < ρ_N ≤ 1.0) out to the separatrix (ρ_N = 1).

To explore the role and importance of the many diverse properties of plasma sources, sinks, and transport involved in the pedestal, a number of different and different types of transport codes will be used. The particular codes involved are: the 1.5D (one-dimensional “radial” transport across shaped flux surfaces) core-based interpretive transport code often used to analyze DIII-D plasmas (ONETWO [5]), a 1.5D pedestal-focused interpretive transport code (GTEDGE [6]), a 1.5D predictive transport code (ASTRA [7]), and two-dimensional (2D) edge-divertor region codes (SOLPS [8], UEDGE [9, 10]) run in interpretive and semi-predictive modes. A key contribution of these codes is that they quantify the various sources (e.g., heating by energetic neutral beams, fueling
Analysis of pedestal plasma transport

by recycling neutrals) and sinks (e.g., radiative energy losses) of density, momentum, and energy in the plasma. This information allows one to determine via “interpretive” transport analysis the plasma transport required to obtain the experimentally-measured density and temperature profiles. Conversely, in “predictive” transport analysis one uses theoretically predicted plasma transport properties to predict the profiles.

The 1.5D codes are applicable for closed, nested flux surfaces inside a divertor separatrix and take account of the non-circular plasma cross section. They average over the flux surfaces to obtain a one-dimensional (1D) description of plasma sources and sinks, and plasma density and energy transport “radially” across toroidal magnetic flux surfaces. The ONETWO and ASTRA codes have been extensively benchmarked against lots of core plasma data and carefully account for plasma sources and sinks in the core plasma. They use a combination of collisional (neoclassical) and (in predictive modes) various analytic-based anomalous plasma transport models. Since the ONETWO code is used to analyze plasma transport in many DIII-D discharges, its results will generally be the “reference” against which other code results will be compared. The GTEDGE code concentrates on the pedestal region using a 1.5D transport model interfaced with results from a 2D kinetic-based neutral source model; it includes consideration of momentum (force balance) equations for each plasma species. The 2D codes (SOLPS, UEDGE) have traditionally focused on modeling divertor plasma properties outside the separatrix. They allow for and determine poloidal variations in plasma sources, sinks and properties. The 2D codes use local poloidal-magnetic-flux-surface-based coordinates that are applicable both inside and outside the separatrix. They determine plasma properties all the way from the core plasma through the pedestal region and scrape-off-layer outside the separatrix, out to vacuum wall and divertor plate boundaries where plasma-wall interactions occur and are included. The 2D codes use the Braginskii [11] collisional plasma transport model with \textit{ad hoc} additions of anomalous radial plasma particle and energy diffusivities. The SOLPS code uses a Monte Carlo-based kinetic neutral model while the UEDGE code uses a fluid neutrals model. A major issue that will be addressed in this paper (mainly in Section 7) is: how far out radially (in the pedestal, toward the separatrix) can 1.5D codes capture the most important transport processes, and where do which 2D processes become important in the pedestal?

This paper is organized as follows. The next section describes the particular DIII-D discharge 98889 being considered and its pedestal plasma properties. The following section describes the transport modeling procedures used by the various codes; mathematical details of the coordinate systems they use are discussed in the Appendix. Section 4 presents the effective diffusivity coefficients for electron and ion heat in this pedestal obtained via interpretive modeling. Comparisons of these effective heat diffusivity coefficients with various analytic-based theoretical predictions are discussed in the following section. The next section describes ASTRA predictive modeling of the pedestal electron temperature profile. Pedestal plasma transport analyses from the 2D codes are discussed in Section 7 which emphasizes the poloidal asymmetry of the recycling neutrals fueling source and discusses the degree to which 1.5D codes
capture the needed physics inside the separatrix. The 1.5D density transport analysis is discussed in Section 8. The next section discusses the new interpretive procedure for determining the density pinch and a possible explanation of it. Section 10 discusses some of the largest uncertainties in the present plasma transport analyses. It also discusses the experimental measurements and modeling extensions most needed to resolve these uncertainties and facilitate better modeling of pedestal plasma transport properties. The final section summarizes the present conclusions of this HEP BE study.

2. Experimental characterization of DIII-D H-mode pedestal

The pedestal to be studied comes from DIII-D discharge 98889, which has been well characterized. Waveforms for various key plasma parameters in this lower single null (LSN) divertor discharge during the time over which averaged profiles are obtained are shown in Fig. 1. As can be ascertained from this figure, during this time window: the ELMs are relatively reproducible; the average time between ELMs is 33.53 ms; and the pedestal electron temperature recovers to its transport quasi-equilibrium value in about 10 ms. Microwave reflectometry data has recently become available on DIII-D. It shows
the density pedestal in discharges similar to 98889 recover on a similar time scale — see Fig. 9 in Ref. [12]. The Thomson scattering data in the bottom trace of Fig. 1 is consistent with such a recovery time, but too widely spaced in time to be definitive.

In order to reduce scatter in the Thomson scattering data and facilitate referencing this data to the separatrix, an averaging procedure was used to obtain composite quasi-equilibrium plasma profiles. The averaging procedure was enabled by the fact that the ELMs were relatively reproducible (see Fig. 1) over the 4 to 5 s time window used. First, pedestal region magnetic flux surfaces were obtained by solving the Grad-Shafranov equation using the EFIT free boundary code [13] for each Thomson time in Fig. 1 allowing for a finite current on the separatrix and using the usual magnetic diagnostics to constrain the EFIT solutions. Previous analyses in DIII-D have identified [14] that the best measure of the separatrix location is where the electron temperature is about 100 eV, as determined by the transition to open field lines outside the separatrix. Also, it was shown [14, 15] that the pedestal profiles can be best fit by hyperbolic tangent (tanh) fitting functions. Thus, next a nonlinear least squares fitter was used to obtain a tanh fit for the Thomson data in 20 % windows of the ELM cycles. Then, the Thomson profile flux coordinate positions were adjusted so the tanh fit $T_e$ on the separatrix is a common, physics-determined value, in this case 90 eV. Further details on the fitting procedure and its uncertainties are provided in Section 2 of Ref. [16].

The resultant composite plasma profiles are then used to develop full “kinetic” EFIT equilibria at a few specific times during the average ELM cycle. These equilibria contain the best available reconstructions of the pressure and current density profiles. An example of flux surfaces from a kinetic EFIT is shown in figure 2. This equilibrium represents the 60-80 % interval of the average ELM cycle and is labeled with a time of 4523 ms, which is 70 % of an average ELM period after the mid-point of the 4 to 5 second averaging time interval. The cross section of the flux surfaces are shown along with the corresponding cylindricalized (see next section and the appendix) flux surface of the separatrix. The toroidal magnetic field at the geometric center of the vacuum vessel at $R_0 = 1.6955$ m is $B_{t0} = 2$ T. For this magnetic equilibrium the magnetic axis is at $R_m \simeq 1.75$ m. The mid-plane half radius $r_M$ is $\simeq 0.6$ m. However, the average radius, which is defined in terms of the toroidal magnetic flux $\Phi$ at the separatrix by $a = \sqrt{\Phi(\text{sep})/\pi B_{t0}}$, is 0.7667 m. It is larger than $r_M$ because the plasma has an edge ellipticity of $\kappa = 1.76$ and triangularity of $\delta = -0.02$. Other parameters during the near steady-state conditions to be analyzed are: plasma current $I = 1.22$ MA and $q_{95} \simeq 4.44$.

Interpretive modeling with ONETWO indicates the total input power was 3.21 MW, from a combination of energetic neutral beams (2.91 MW of which 1.17 MW goes to electrons, 1.74 MW to ions) and ohmic heating (0.3 MW). This total input power is about a factor of 4 higher than the L-H transition power threshold of 0.75 MW determined from the latest ITER scaling [17] at an earlier time when the density is lower. Global plasma parameters are: central electron temperature $T_e(0) = 3.2$ keV and deuterium ion temperature $T_i(0) = 4.6$ keV, toroidal beta $\beta_t = 0.013$, poloidal beta $\beta_p = 0.59$, thermal energy confinement time $\tau_E \simeq 0.153$ s $[\text{H ITER98y2} \simeq 1.02]$, and
estimated particle confinement time $\tau_p \simeq 0.4$ s.

The composite profiles of the electron density and temperature, and ion temperature are shown in Fig. 3. Data for the electron density $n_e$ and temperature $T_e$ were obtained from a multi-point, multi-time Thomson scattering (TS) system [18], which has a high density of viewing chords in the pedestal region. The edge channels view along the vertical Thomson laser; they are separated by 1.3 cm (about 0.013 in $\rho_N$) with the spot size being comparable to the separation. Thomson profile measurements were made every 12–13 ms throughout the discharge considered here. Data for the ion temperature $T_i$ were obtained from a charge exchange recombination (CER) spectroscopy system, which views the C VI 5290.5 Å line [19]. This system also has a high density of view chords in the pedestal region with a separation of tangential chords, oriented along the outer mid-plane, of about 0.6 cm (about 0.01 in $\rho_N$) and a spot size slightly smaller than the chord separation. A system of vertically viewing chords is interleaved with the edge chords and is often used to improve the spatial resolution for $T_i$. The $T_i$ data were acquired with an averaging time of 10 ms. The TS tanh fits at the separatrix
Figure 3. Edge profiles of: a) electron density \( n_e/(10^{20} \text{ m}^{-3}) \), b) electron temperature \( T_e(\text{keV}) \), and c) ion temperature \( T_i(\text{keV}) \). The fundamental normalized radial coordinate used here, \( \rho_N \equiv \sqrt{\Phi/\pi B_0/a} \), is based on the toroidal magnetic flux \( \Phi \). Data used in obtaining the tanh fits (solid lines) are indicated by plus (+) symbols, which illustrate the degree of uncertainty in these profiles and the tanh function fits to them. The tanh \( \rho_N \) symmetry points are indicated by large dots (•) on the tanh fit lines. The edge regions identified here are: I core, II top and III bottom halves of the pedestal. The normalized poloidal flux coordinate \( \Psi_N \) and radial distance along the horizontal outboard mid-plane \( R - R_{\text{sep}} \) (m) are indicated in rulers at the bottom.

\( (\rho_N = 1) \) yield \( n_e(1) \simeq 0.077 \times 10^{20} \text{ m}^{-3} \) and \( T_e(1) \simeq 0.09 \text{ keV} \). At the \( \rho_N \) symmetry points of the tanh fits \( n_e(0.982) \simeq 0.165 \times 10^{20} \text{ m}^{-3} \) and \( T_e(0.978) \simeq 0.39 \text{ keV} \). Spline fits to the CER-determined ion temperature profiles were used in the various modeling codes, which yield \( T_i(1) \simeq 0.28 \text{ keV} \) and \( T_i(0.982) \simeq 0.49 \text{ keV} \).

In Fig. 3 and throughout the remainder of this paper we identify three key regions (I, II, III) of the edge plasma being analyzed. There is no unique definition for the position of the “top” of the pedestal. Here, we will define it to occur at \( \rho_N \simeq 0.94 \),
which is about twice the width of region III. Region III is defined to be from the symmetry points \( \rho_N \simeq 0.98 \) of the pedestal tanh fits to the separatrix. Thus, region I will be the extension of the core plasma region out to the top of the pedestal, i.e., here \( 0.85 < \rho_N < 0.94 \). Region II will be called the “top half” of the pedestal and is defined by \( 0.94 < \rho_N < 0.98 \). Finally, region III will be called the “bottom half” of the pedestal and is defined by \( 0.98 < \rho_N < 1.0 \).

3. Transport modeling equations and procedures

The fundamental plasma transport equations solved by all the codes used in this study are developed from the collisional Braginskii [11] density and energy equations for each species of charged particles in the plasma, which will be written initially in the form

\[
\text{density: } \frac{\partial n}{\partial t} + \nabla \cdot n \mathbf{V} = S_n, \tag{1}
\]

\[
\text{energy: } \frac{\partial}{\partial t} \left( \frac{3}{2} nT \right) + \nabla \cdot \left( q + \frac{5}{2} nT \mathbf{V} \right) = Q. \tag{2}
\]

Here, \( n \) is the density (units of \( \text{m}^{-3} \)), \( \mathbf{V} \) is the flow velocity (m/s), and \( T \) is the temperature (eV) of the species being considered. The density source \( S_n \) (m\(^{-3}\)s\(^{-1}\)) has been added to the usual Braginskii density equation to allow for particle sources and sinks. It is predominantly due to ionization of recycling thermal neutrals emanating from the LSN divertor X-point region in DIII-D discharge 98889 — see Section 6. (The net beam source of ions from ionization minus charge exchange of beam neutrals is only 10% of the recycling neutrals ionization source at \( \rho_N = 0.85 \) and is negligible in the pedestal region \( 0.94 \leq \rho_N \leq 1.0 \).) The GTEDGE code [6] solves in addition various components of the species momentum (force balance) equation for the flow velocity components; these solutions and their consequences will be discussed in Section 9.

In the energy equation, \( (3/2) nT \) is the internal energy (eV/m\(^3\)), \( q \) is the conductive heat flux (W/m\(^2\)) and \( (5/2) nT \mathbf{V} \) is the convective heat flux (W/m\(^2\)). The \( Q \) (W/m\(^3\)) on the right side of the energy equation represents the net energy input to or loss from the species due to cross-species collisional energy exchange, joule heating, and energy sources and sinks; it will be discussed in the next paragraph. In ONETWO [5] and SOLPS [8] the ion flow energy \((1/2) m_i n_i |\mathbf{V}_i|^2\) is added to the ion internal energy and convective heat transport; this flow energy will be neglected because it gives only a very small modification (\( \lesssim 1\% \)) to the internal energy in the 98889 pedestal.

The pedestal heating (or energy loss) rates per unit volume (power density) \( Q \) of electrons and ions have many components. The dominant ones from 1.5D ONETWO modeling are shown in Fig. 4. In the pedestal region \( 0.94 \leq \rho_N \leq 1 \) (II and III), Fig. 4a shows that electrons are heated modestly by the collisional energy exchange \( Q_\Delta \) (because \( T_i > T_e \) in the pedestal), with smaller contributions due to joule heating \( Q_{\text{OH}} \) and (nearly negligible) neutral beam heating \( Q_{e\text{beam}} \). Radiative losses \( Q_{\text{rad}} \) are a significant but not dominant electron heat loss process in the pedestal. The radiated power is measured by a multi-chord, multi-channel bolometer array; the total radiated
The dominant ion energy losses in the pedestal region 0.94 \( \leq \rho_N \leq 1 \) (II and III) are more significant. They can be seen from Fig. 4b to be mainly due to charge-exchange and collisional energy transfer \( Q_\Delta \) to the colder electrons. There is modest heating from the production of ions via electron impact ionization of recycling neutrals \( Q_{i_{\text{ion}}} \).

Similar dominant contributions have been found in modeling of other DIII-D pedestals [20]–[23]. However, electron heating by neutral beams can be larger when the beam power and pedestal density are higher [20]–[22]. There can also be larger charge exchange ion energy losses when the ion temperature is higher near the separatrix [22].

Because the various transport modeling codes focus on different regions of the plasma, they use different coordinate systems. All the codes assume axisymmetry in the toroidal (angle \( \zeta \)) direction. The 1.5D core transport codes (ONETWO, ASTRA) use magnetic flux coordinates based on the nested toroidal magnetic flux surfaces (toroidal flux \( \psi_t \rightarrow \Phi \) in ONETWO, ASTRA). For their radial coordinate they use...
\[ \rho \equiv (\psi_t / \pi B_0)^{1/2}, \] which is the average radius of the \( \psi_t \) or \( \Phi \) toroidal flux surface. In this definition \( B_0 \) is the magnitude of the toroidal magnetic field at the geometric axis of the vacuum vessel (i.e., at \( R_0 = 1.6955 \) m). The core transport codes base their coordinate system on the toroidal magnetic flux because during typical magnetic field transients (e.g., due to a non-inductive current drive being turned on) the toroidal magnetic field (flux \( \psi_t \)) changes (in magnitude and spatial structure) much less than the poloidal magnetic field (flux \( \psi_p \)), which is determined by the toroidal current distribution in the plasma. However, the toroidal flux \( \psi_t \) or \( \Phi \) is not well-defined outside the separatrix — because the area involved in its definition extends vertically through the vacuum vessel coils, past the divertor coils, and vertically to infinity.

The 2D codes (SOLPS and UEDGE), which focus on edge plasmas up to divertor plate and wall boundaries, use a poloidal-flux-based radial coordinate, which is well-defined all the way to divertor plates. In ONETWO the poloidal flux \( \psi_p \rightarrow \Psi \) and a corresponding normalized radial coordinate \( \Psi_N \equiv \Psi/\Psi_{sep} \) are often used. Both the toroidal and poloidal flux coordinates are applicable in the pedestal region in the quasi-equilibrium transport situation being studied here. The 2D codes often present results in terms of the radial distance in from the separatrix on the horizontal mid-plane of the plasma cross-section, \( R - R_{sep} \).

The rulers at the bottom of Fig. 3 show the relations between these three different radial coordinate systems in the plasma edge. This paper will present results primarily in terms of the normalized toroidal-flux-based radial coordinate \( \rho_N \equiv \rho/a \), in which \( \rho \) is the average minor radius of the flux surface and \( a \equiv (\psi_{sep}/\pi B_0)^{1/2} \) is the average minor radius of the separatrix (see Fig. 2).

We will be mainly interested in net plasma transport radially from one flux surface to the next. Thus, it is convenient to define the flux-surface average (FSA) of an axisymmetric (i.e., \( \partial f / \partial \zeta = 0 \), in which \( \zeta \) is the toroidal angle) scalar function \( f(x_\psi, x_\theta) \):

\[ \langle f \rangle \equiv \lim_{\delta x_\psi \to 0} \frac{\int_{x_\psi}^{x_\psi + \delta x_\psi} f \, d^3x}{\int_{x_\psi}^{x_\psi + \delta x_\psi} d^3x} = \frac{2 \pi \int dx_\theta \sqrt{g} f}{dV/dx_\psi}. \] (3)

Here, as discussed in the Appendix, \( x_\psi \) is a generic radial flux-surface-based coordinate and \( x_\theta \) is a generic poloidal coordinate on a flux surface. Also, \( \sqrt{g} \equiv 1/(\nabla x_\psi \cdot \nabla x_\theta \times \nabla \zeta) = (d\psi_p/dx_\psi)/B \cdot \nabla x_\theta \) is the Jacobian of the transformation from the laboratory to generic \((x_\psi, x_\theta, \zeta)\) flux-surface-based coordinates and \( \int dx_\theta \) indicates integration over the cyclic generic poloidal coordinate \( x_\theta \) on a \( x_\psi \) flux surface. Also,

\[ dV/dx_\psi \equiv V' = 2\pi \int dx_\theta \sqrt{g} \] (4)
is the radial derivative of the volume \( V(x_\psi) \equiv \int_0^{x_\psi} d^3x \) encompassed by the flux surface labeled by \( x_\psi \). Note that \( V' \) has units of volume (m\(^3\)) divided by the units of \( x_\psi \). The FSA of the divergence of a vector such as the particle flux \( \Gamma \) is

\[ \langle \nabla \cdot \Gamma \rangle \equiv \frac{\partial}{\partial V'} \langle \Gamma \cdot \nabla V \rangle = \frac{1}{V'} \frac{\partial}{\partial x_\psi} (V' \langle \Gamma \cdot \nabla x_\psi \rangle). \] (5)
The FSA annihilates the poloidal \((x_\theta)\) derivative terms implicit in (1), (2) which are specified in (A.3), (A.4). Thus, the FSA density and energy equations become

\[
\frac{\partial \langle n \rangle}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial x_\psi} \left( V' (\Gamma \cdot \nabla x_\psi) \right) = \langle S_n \rangle,
\]

\[
\frac{\partial}{\partial t} \left( \frac{3}{2} \langle n T \rangle \right) + \frac{1}{V'} \frac{\partial}{\partial x_\psi} \left( V' \langle (q + \frac{5}{2} \mathbf{TT}) \cdot \nabla x_\psi \rangle \right) = \langle Q \rangle.
\]

These are the one-dimensional transport equations solved in the 1.5D modeling codes.

4. Interpretive 1.5D modeling of heat transport

In general the radial fluxes \(\Gamma\) and \(q\) vary poloidally on a flux surface. However, in the core- and pedestal-relevant low (banana-plateau) collisionality regime where collision lengths are long compared to the poloidal periodicity length along magnetic field lines, net radial (i.e., across field lines) plasma transport from one flux surface to the next is the only physically meaningful quantity. In terms of the generic coordinates, the flow of a vector particle flux \(\Gamma\) across a flux surface is defined by

\[
\int dS(x_\psi) \cdot \Gamma = 2\pi \int d\theta \sqrt{g} \Gamma \cdot \nabla x_\psi = (dV/dx_\psi) \langle \Gamma \cdot \nabla x_\psi \rangle = S(x_\psi) \langle \Gamma \cdot \nabla x_\psi \rangle / |\nabla x_\psi|.
\]

In obtaining the last forms we used \(dS(x_\psi) \equiv 2\pi \sqrt{g} dx_\psi \nabla x_\psi = 2\pi \sqrt{g} dx_\psi |\nabla x_\psi| \hat{e}_{x_\psi},\) which can be used to show that the area of a flux surface is \(S(x_\psi) = |\nabla x_\psi| \, dV/dx_\psi.\) Thus, the physically relevant net particle flow and electron and ion conductive heat flows (energy flowing) across a flux surface are

\[
\dot{N}(x_\psi) \equiv \int dS(x_\psi) \cdot \Gamma = V' \langle \Gamma \cdot \nabla x_\psi \rangle, \quad \text{s}^{-1},
\]

\[
P_{\text{cond}}(x_\psi) \equiv \int dS(x_\psi) \cdot q = V' \langle q \cdot \nabla x_\psi \rangle, \quad \text{Watts}.
\]

As a check that all the modeling codes are beginning with the same “input data,” in Fig. 5 we show the heat flows defined in (9) for the edge region. Here, the convective heat flow is defined by \(P_{\text{conv}} \equiv f dS(x_\psi) \cdot (5/2) nT \mathbf{V} = V' \langle (5/2) \mathbf{TT} \cdot \nabla x_\psi \rangle.\) (The 5/2 factor in the convective heat flow will be used throughout this paper since it arises naturally in the collisional Braginskii [11] fluid moment equations and is what most of these modeling codes usually use.) The conductive power \(P_{\text{cond}}\) will be defined as the total power flow \(P_{\text{tot}}\) minus the convective heat flow \(P_{\text{conv}}\) through the surface: \(P_{\text{cond}} \equiv P_{\text{tot}} - P_{\text{conv}}.\) It is an experimentally-inferred quantity; i.e., it does not result from or necessarily imply the Fourier heat flux form [see Eq. (11) below] used below in interpretive analyses.

As can be seen in Fig. 5, the various modeling codes agree reasonably well on the heat flows through the pedestal region. The total electron power flow \((\sim 1.8 \text{ MW})\) through the pedestal is nearly constant with \(\rho_N\) — because the net electron heating in this region (II and III) is rather small (see Fig. 4a). The total ion power flow \((\sim 0.7 \text{ MW})\) through the pedestal decreases somewhat with increasing \(\rho_N\) because of collisional energy losses to the colder electrons and charge exchange losses there (see Fig. 4b). The net ion cooling power in the pedestal region (II and III, i.e., \(0.94 \leq \rho_N \leq 1\)), which has a volume \(\delta V \simeq 1.5 \text{ m}^3,\) is \(\delta P_i \simeq \delta V Q_i \sim (1.5 \text{ m}^3) \left(10^5 \text{ W/m}^3\right) \sim -0.15\)
Analysis of pedestal plasma transport

Figure 5. Heat flows through pedestal flux surfaces obtained from various modeling codes: a) electron total and convective heat flows (MW), and b) ion total and convective heat flows (MW).

MW. Since variations in heat flows through the pedestal region are relatively small fractions of net energy flows there, the energy balances in this pedestal are dominated by the heat flows through the pedestal region, not by local heating or cooling processes there. However, the ion cooling processes are significant in the pedestal and become progressively more important as one approaches the separatrix (cf., Fig. 5b). Previous transport analyses [20, 21, 22] of radial heat flows in similar DIII-D pedestals have come to similar conclusions, except for cases where larger charge exchange ion heat losses near the separatrix strongly reduce the conductive ion heat flow [20, 21].

Figure 5 also shows that the convective heat flow is relatively small for the pedestal we are considering; it is largest near the separatrix where it is about 10% of the total electron heat flow and about 25% of the total ion heat flow. The variability in modeling code results for convective heat flows near the separatrix is caused mainly by differences in models of the plasma fueling by recycling thermal neutrals. Section 7 will discuss the neutral models used in the various codes; in particular, the 2D neutral models and their effects on the particle flow \( \dot{N} \) defined in (8) are discussed there. The density transport properties in the pedestal are discussed in Sections 8 and 9.

Plasma transport (caused by collisions and microturbulence) is often characterized by effective diffusivity coefficients for particle and heat transport. Specifically, in analogy with the kinetic theory of gases, one assumes the particle flux is given by a Fick’s
diffusion law $\Gamma = -D \nabla n$ and the conductive heat fluxes are in the Fourier heat flux form $q = -n \chi \nabla T$. The effective diffusion coefficients $D$ and $\chi$, which have units of $m^2/s$, are determined “interpretively” from experimental data by dividing the transport fluxes by the appropriate plasma gradients. While this procedure neglects possible “off-diagonal” transport fluxes (e.g., particle fluxes induced by temperature gradients) and possible “pinch” flux (or threshold gradient) effects, it does provide a measure of the transport required by a dominantly diffusive process to yield a given transport flux in response to the respective plasma gradient. Plausible pinch flow effects in density transport will be discussed in Sections 8 and 9 below.

Plasmas in the hot core of tokamaks often hover near the threshold for onset of drift-wave-type microinstabilities. Then, small changes in temperature or density gradients can cause very large changes in transport fluxes. For such cases the recommended procedure [25] for analyzing plasma transport is to use theoretical or computational predictions of the (highly nonlinear) transport fluxes $\Gamma$ and $q$ in Eqs. (6) and (7) to determine the density and temperature profiles. Unfortunately, this procedure is not useful here, mainly because, as discussed in Sections 5 and 6, most drift-wave-type modes are stable and/or cause little or no radial plasma transport in the pedestal region, especially in its bottom half (III). Recently, MHD-like kinetic ballooning modes (KBMs) have been advanced [26] as a similar possible determiner of the pedestal pressure profile gradient; however, the nonlinear evolution (turbulence or intermittency?) of KBMs and possible transport representation of their effects have not yet been developed. Here, we will use the original, more primitive procedure of interpreting effective heat and particle diffusivities from the ratio of transport fluxes to plasma gradients, which gives at least an initial perspective on the relative amounts of plasma transport taking place in the various electron and ion density and heat transport channels.

Further assumptions are needed to facilitate comparisons of radial transport in 1.5D and 2D codes. First, one usually assumes the plasma density and temperatures are approximately constant on flux surfaces; Section 7 will show that this is the case for the 98889 DIII-D pedestal. Next, the diffusivities are assumed to be constant on flux surfaces. Physically, diffusivities and transport fluxes vary significantly on flux surfaces; they usually peak strongly on the tokamak’s outboard side where particle gyroradii and drift orbit excursions from flux surfaces are largest and microinstabilities “balloon.” However, characterizing radial transport by diffusion coefficients that are constant on flux surfaces is reasonable in the low collisionality regimes of interest here where the collision lengths exceed the poloidal periodicity length which causes the only relevant transport to be from one magnetic flux surface to the next. Thus, labeling flux surfaces for simplicity as in the ONETWO and ASTRA codes by $x_\psi \rightarrow \rho$, the Fick’s diffusion and Fourier heat flux laws become

$$\Gamma = - D(\rho) \nabla n = - D \nabla_\rho dn/d\rho, \quad (10)$$
$$q = - n \chi(\rho) \nabla T = - n \chi \nabla_\rho dT/d\rho. \quad (11)$$

The $\nabla_\rho$ factors represent the poloidal variation on a magnetic flux surface of the
Analysis of pedestal plasma transport

Figure 6. Poloidal variation of radial electron heat flux ($\propto |\nabla \rho|$) at the edge of the core plasma for $\chi_e$ constant on a flux surface: from UEDGE and SOLPS modeling and from estimates of $|\nabla \rho|$ (dots, squares) using the analytic Miller geometry model [27]. [Reprinted from Ref. [28], W.M. Stacey, Phys. Plasmas 15, 122505 (2008). Copyright © 2008 American Institute of Physics.]

“radial” gradients (in laboratory coordinates) of density and temperatures from one flux surface to the next (see Fig. 2). The resultant poloidal variation ($\propto |\nabla \rho|$) on flux surfaces of the electron heat flux from (11) is illustrated in Fig. 6 at $\rho_N \approx 0.82$ ($R_{sep} - R \approx 6$ cm) in 98889. The heat flux is largest on the outboard mid-plane where the magnetic flux surfaces are most closely bunched together (compressed) and the local temperature gradients are largest. The smallest radial heat fluxes occur at the top and bottom (near the LSN divertor X-point) of the tokamak where the flux surfaces are most widely separated and local radial gradients are smallest. In reality, as was explained in the preceding paragraph, the heat fluxes are even more strongly peaked on the outboard mid-plane than is indicated by the Fourier heat flux model in (11) with constant heat diffusivity on a magnetic flux surface that is being used in these codes.

Substituting the expressions in (10) and (11) into (8) and (9) yields

$$D = \frac{\dot{N}(\rho)}{(dV/d\rho) \langle |\nabla \rho|^2 \rangle (-dn/d\rho)},$$  \hspace{1cm} (12)

$$\chi = \frac{P_{\text{cond}}(\rho)}{(dV/d\rho) \langle |\nabla \rho|^2 \rangle (-n \,dT/d\rho)},$$  \hspace{1cm} (13)

Using these formulas to determine effective particle and heat diffusivities is usually
referred to as interpretive transport modeling. From (3) and the definition of the surface area above (8) we see that the volume factor \( dV/d\rho = S(\rho)/\langle|\nabla \rho|\rangle \), in which \( S(\rho) \) is the area of the \( \rho \) flux surface. For reference, in the pedestal region \( 0.94 \leq \rho_N \leq 1 \), ONETWO indicates \( S \simeq 53 \rho_N \text{ m}^2 \), \( \langle|\nabla \rho|\rangle \simeq 1.22 \), and \( dV/d\rho \simeq 43.4 \rho_N \text{ m}^2 \).

The dimensionless factor \( \langle|\nabla \rho|^2\rangle \) in (12) and (13) is the flux surface average of the square of the poloidal variation of the heat flux (\( \propto |\nabla \rho| \)) illustrated in Fig. 6. It is near unity in the core of tokamak plasmas; for example, in 98889 it is about 1.15 at \( \rho_N \simeq 0.5 \) (the half-radius of the plasma). Thus, it is often neglected in core transport studies [29]. However, as shown in Fig. 7, in the pedestal region (II and III) it is of order a factor of two and varies modestly with \( \rho_N \) there; hence it must be taken into account. For example, the diffusivity values quoted in the ONETWO output files are in fact \( \langle|\nabla \rho|^2\rangle \chi \) and must be divided by \( \langle|\nabla \rho|^2\rangle \) to yield the \( \chi \) values to be compared with other transport modeling codes and theoretical formulas for \( \chi(\rho) \).

The interpretive, effective electron and ion heat diffusivities determined from (13) using the conductive heat flows \( P_{\text{cond}} = P_{\text{tot}} - P_{\text{conv}} \) from Fig. 5 and the \( \langle|\nabla \rho|^2\rangle \) factor from Fig. 7 are shown in Fig. 8. Given the wide variety of physics models, coordinate systems, and numerical procedures used in the various modeling codes run in their interpretive modes, the degree of agreement between them is satisfactory. Since all the interpretive codes begin from the same plasma profile gradients, differences in these effective diffusivities are primarily due to minor differences in the computed heat flows.
through various regions of the pedestal. Averaging the interpretive results, the minimum heat diffusivities for the 98889 pedestal are about 0.3 m$^2$/s for electrons and 0.15 m$^2$/s for ions. The interpretive diffusivities near the minima agree within about 25 % for $\chi_e$ and 50 % for $\chi_i$. The interpretive ion heat diffusivities vary the most mainly because of the differences in the charge exchange ion heat losses in the pedestal, as indicated in Figs. 4b and 5b. The different charge exchange losses result from differences in recycling neutrals fueling models, which are discussed at the end of Section 7. The variations in modeling code results are comparable to the systematic and statistical experimental uncertainties in the gradients obtained from the tanh fits of the $n_e$ and $T_e$, and spline fit $T_i$ profiles (see Fig. 3) used in these transport analyses.

The strong radial variations of these interpretive, effective heat diffusivities in the pedestal region are generic. The reasons for them can be understood by examining the role of the various terms in (13). As noted above, $P_{\text{cond}}$, $V'$, and $\langle |\nabla \rho|^2 \rangle$ don’t vary much in the pedestal region. Thus, variations in the effective heat diffusivities are caused mainly by changes in the magnitudes of the temperature gradients and the density. As $\rho_N$ increases from the core into the pedestal, the effective heat diffusivities first decrease, mainly because the temperature gradients are increasing as $\rho_N$ increases toward and down the top half (II) of the pedestal. The diffusivities reach a minimum a bit before the extrema of the temperature gradients (at symmetry points of the tanh
fits at $\rho_N \simeq 0.98$); these minima represent the H-mode pedestal “transport barriers” in electron and ion heat transport. Finally, the interpretive $\chi$ values increase in the bottom half (III) of the pedestal as $\rho_N$ increases further outward toward the separatrix, mainly because the density decreases significantly there; the decreasing magnitudes of the temperature gradients there also contribute slightly.

The specific $\chi$ profiles obtained in Fig. 8 result from the tanh and spline fits to the data used to represent the input data for the modeling codes. However, similar $\chi_e$ and $\chi_i$ profiles have been obtained from interpretive SOLPS transport modeling of some ASDEX-U H-mode pedestals [30, 31]. Also, GTEDGE modeling of other DIII-D H-mode pedestals [20][23] has obtained similar profiles.

5. Comparisons of heat diffusivities with theoretical predictions

Radial profiles of some theoretically-relevant magnetic field structure and plasma transport properties in the edge region are shown in Figs. 9 and 10. These parameters are obtained from ONETWO modeling using the kinetic EFIT equilibrium flux surfaces.
in Fig. 2 and the fitted plasma profiles shown in Fig. 3. Note that the \( q \) profile in Fig. 9a has a slight negative slope in the region \( 0.955 \lesssim \rho_N \lesssim 0.97 \), which causes the global magnetic shear parameter \( \delta \equiv (\rho_N/q)(d\rho/d\rho_N) \) to be negative there. This effect is caused by the increase of the bootstrap current density to about 4.6 times the ohmic current density as \( \rho_N \) increases toward the symmetry point of the pedestal (\( \rho_{N\text{sym}} \approx 0.98 \)).

As Fig. 9b shows, in the pedestal region the gradient scale lengths are small fractions of the average plasma radius \( a \). The pedestal width \( \Delta \) [4, 14, 15] is about twice the normalized gradient scale length at the symmetry point: \( \Delta \rho_e \approx 0.047 \sim 2L_{ne}(\rho_{N\text{sym}})/a \), \( \Delta_{Te} \approx 0.043 \sim 2L_{Te}(\rho_{N\text{sym}})/a \) and \( \Delta_{Ti} \approx 0.106 \sim 2L_{Ti}(\rho_{N\text{sym}})/a \). Because the ratio of the deuterium ion gyroradius to the density gradient scale length is of order 0.1 or smaller, a small gyroradius expansion is valid throughout the pedestal. But since the half-width of banana drift orbits are a factor of order \( 2 \) larger than the ion gyroradius in the pedestal region, there could be significant ion orbit losses from the bottom half (III) of the pedestal. However, since thermal deuterium ions are near the transition from the banana to plateau collisionality regime there (see Fig. 10 and next paragraph), it is likely that only superthermal ions on the tail of the deuterium ion distribution will be lost from this region. Fully ionized carbon ions have smaller gyroradii than deuterium ions by a factor of about \( 1/\sqrt{6} \approx 0.4 \); they are also more collisional by a factor of \( Z_i^2 \approx 36 \). Thus, apparently only “high energy tail” fully stripped carbon ions will contribute to the CER-inferred carbon ion temperatures measured outside the separatrix, as indicated in Fig. 3c.

As indicated in Fig. 10a, in the core (I) and top half (II) of the edge plasma, thermal electrons and ions are both in the banana collisionality regime \( (\nu_{ss} \ll 1, \nu_{ss} \equiv \nu_s/(\epsilon^{3/2}v_{Ts}/R_0q)) \), which is \( \sqrt{2} \) smaller than the definition in [32]). Fully stripped carbon impurities \( (n_C/n_e \approx 0.061) \) in the edge region are the only significant impurity in DIII-D plasmas. They have been taken into account in the electron and ion collision frequencies via \( \nu_e \propto Z_i \text{eff} \) and \( \nu_i \propto [1 + \sqrt{2}(n_eZ_i \text{eff}/n_i - 1)]/\sqrt{2} \) with \( Z_i \text{eff} \equiv \sum_i n_i Z_i^2/n_e = (n_i + n_C Z_C^2)/n_e \approx 2.83 \) throughout the edge region being considered. Also, \( v_{Ts} \equiv (2T_e/m_a)^{1/2} \) and \( \epsilon \) is an inverse aspect ratio parameter that represents the variation of the magnetic field strength on a flux surface: \( \epsilon \equiv (B_{max} - B_{\text{min}})/(B_{max} + B_{\text{min}}) \approx r_M/R_m \), which is \( \approx 0.6/1.75 \approx 0.34 \) in the pedestal region. Electrons are in the plateau regime \( (1 < \nu_e < \epsilon^{-3/2} \approx 5) \) in the bottom half (III) of the pedestal and barely reach the Pfirsch-Schlüter regime \( (\nu_e > \epsilon^{-3/2} \approx 5) \) just inside the separatrix (i.e., for \( \rho_N > 0.995 \)). Deuterium thermal ions are near the banana-plateau transition all the way to the separatrix — because \( T_i \) does not decrease much in the bottom half (III) of the pedestal.

The parameters \( \eta_t \) (\( > \eta_{t,\text{crit}} \approx 1.25 \) [33]), \( \eta_e \) (\( > \eta_{e,\text{crit}} \approx 1.2 \) [34]) and \( R_0/L_T = (R_0/a)(a/L_T) \approx 2.3\,(a/L_T) \) (\( \gtrsim 10 \) for \( \rho_N > 0.91 \)) are all sufficiently large in the core region that ion and electron temperature-gradient-driven drift-wave-type microinstabilities are probable there. However, ion-temperature-gradient (ITG) modes should be stable in most of the pedestal (II and III) because \( \eta_t < 1.25 \) there. Dissipative trapped electron modes (TEMs) are likely to be stabilized by the \( \mathbf{E} \times \mathbf{B} \) flow shear [35] in the pedestal region and by the high (plateau) electron collisionality in the bottom
Analysis of pedestal plasma transport

Figure 10. Radial profiles using ONETWO parameters of: a) neoclassical collisionality parameter [32] for electrons \((\nu_{e*})\) and ions \((\nu_{i*})\); and b) ratios of gradient scale lengths, \(\eta_e \equiv L_{ne}/L_{Te}, \eta_i \equiv L_{ni}/L_{Ti}\).

half (III) of the pedestal. The shorter wavelength and higher growth rate electron-temperature-gradient (ETG) instabilities are less likely to be stabilized by the flow shear [34]. Finally, the negative global magnetic shear \(\hat{s}\) in the region \(0.955 \lesssim \rho_N \lesssim 0.97\) can have a stabilizing effect on these drift-wave-type microinstabilities. However, the local magnetic shear on the outboard side of divertor plasmas is almost always negative in the pedestal region (II and III) and ETG modes are still unstable there [34].

For a generic scaling perspective on possible plasma transport processes, the radial profiles of some characteristic plasma diffusion coefficients are shown in Fig. 11. ETG-induced electron heat transport has been advanced [34] as being a significant contributor to electron heat transport in the pedestal; for example, simulation of an ASDEX-U pedestal at a point where \(T_e \simeq 0.69\) keV predicted \(\chi_e \simeq 0.83 \text{ m}^2/\text{s}\) [34]. In addition, paleoclassical electron heat transport [36, 37] has been advanced as being a major contributor to electron heat diffusion in a different DIII-D pedestal — see Fig. 14 in Ref. [36]. Further, predictive modeling with ASTRA [38] found that for 15 DIII-D H-mode discharges the plasma \(T_e\) profiles were best modeled by a combination of electron heat diffusion from ETG-induced anomalous transport near the top of the pedestal (I to II) and paleoclassical transport throughout the pedestal (II and III). Profiles of the underlying characteristic electron gyro-Bohm and magnetic field diffusivities for these two processes are shown in Fig. 11. From these profiles, we see that ETG-induced
transport is a good candidate to dominate near the top of the pedestal (I to II) — because it scales as $T_e^{3/2}$ and $T_e$ is highest there. In the bottom half (III) of the pedestal paleoclassical electron heat transport is a good candidate to dominate — because there the helical multiplier [36] $M < 1$ so it scales as $T_e^{-3/2}$ and $T_e$ is smallest there and decreases as one moves radially outward toward the separatrix. Also shown for reference is the predicted neoclassical ion heat diffusivity [32] profile obtained from ONETWO using the Chang-Hinton formula [39].

The magnitude and profile of the sum of the generic predictions for ETG-driven gyro-Bohm level and paleoclassical transport shown in Fig. 11 are similar to the interpretive effective electron heat diffusivity $\chi_e$ shown in Fig. 8a. The neoclassical prediction indicated in Fig. 11 is of the same order of magnitude as the experimentally-inferred ion heat diffusivity $\chi_i$ shown in Fig. 8b; however, its profile is quite different and it is about a factor of four larger near the minimum in the interpretive $\chi_i$. Recent calculations with the more precise NEO code [40] have indicated the Chang-Hinton prediction is too large by about 20 % in the plasma core; for a typical pedestal the NEO code indicates [41] the true neoclassical $\chi_i$ is about 30 % smaller than the Chang-Hinton prediction. These calculations include the orbit squeezing effects [42] which are caused...
by the strong radial variation of the radial electric field in the pedestal. Decreasing the Chang-Hinton prediction by 30 %, it still seems that the inferred effective ion heat diffusivity in the 98889 pedestal is less than (by a factor \( \sim 3 \)) the best estimate of the neoclassical ion heat diffusivity there.

The interpretive heat diffusivities have been compared with a wide variety of analytic-based, mixing-length-type theoretical predictions for a number of DIII-D pedestals in Ref. [20]–[23], and most extensively in Ref. [43]. For the pedestal in 98889, Figs. 12 and 13 compare predictions of a number of theoretical models with the interpretive GTEDGE \( \chi_e \) and \( \chi_i \) profiles shown in Fig. 8. Detailed formulas for the various analytic-based theoretical model predictions are given in [43]. To determine radial heat fluxes the GTEDGE interpretive transport model integrates the equilibrium heat transport equations (7) from the separatrix inward using experimentally-determined density and temperature gradients. For this GTEDGE modeling it is assumed that the ratio of ion to electron power flow through the separatrix is 25 % to 75 %. This is quite close to and between the values of this parameter from ONETWO (28/72) and SOLPS (24/76). The data in Figs. 12 and 13 are obtained at a slightly different time slice (\( \sim 3962 \) ms, at the first of the time interval being considered).
Figure 13. Comparison of GTEDGE-determined effective ion heat diffusivity $\chi_i$ (exp) with analytic-based predictions [43] of various theoretical models: neoclassical collision-induced ion heat diffusivity (neocl), and ion-temperature-gradient (itg) and drift-Alfvén (da) microturbulence-induced anomalous ion heat transport.

Various points can be made about the theory-experiment electron heat diffusivity comparisons shown in Fig. 12. As can be seen, the paleoclassical prediction parallels the interpretive, effective $\chi_e(\rho_N)$ in the bottom half of the pedestal (III, $0.98 \leq \rho_N \leq 1.0$) and in the core region (I, $\rho_N \leq 0.94$). There are no points indicated between these regions because integrating as in Eqs. (35) and (36) in Ref. [43] from the separatrix region inward gives negative values for $\chi_e$ in this region (II) and the integration outward from the core becomes inappropriate in this region (II). The four paleoclassical $\chi_e$ values in the bottom half (III) of the pedestal, where the paleoclassical helical multiplier $M < 1$, have been estimated in Fig. 12 by Eq. (32) in Ref. [43].

The spatial variation of the electron-temperature-gradient (ETG) and trapped-electron-mode (TEM) analytic-based predictions, which include [43] the reductions in growth rates caused by $\mathbf{E} \times \mathbf{B}$ flow shear effects, also parallel the interpretive, effective $\chi_e(\rho_N)$ in the core region, but have different magnitudes. In the transition from the core region to the top half of the pedestal they first increase with $\rho_N$ (specifically, for $0.92 \lesssim \rho_N \lesssim 0.95$) — because the analytic-based theoretical formulas in Eqs. (39) and
Analysis of pedestal plasma transport

(43) for ETG and TEM transport in [43] are proportional to the magnitude of the electron temperature gradient which increases with $\rho_N$ there. However, this trend is opposite to the interpretive $\chi_e(\rho_N)$ in this region. The ETG and TEM predictions then decrease rapidly with $\rho_N$ down the top half of the pedestal ($0.95 < \rho_N < 0.98$). In the bottom half of the pedestal electrons are in the plateau collisionality regime (i.e., $\nu_{se} > 1$, see Fig. 10a); hence TEM modes are stable there. Finally, the ETG $\chi_e$ increases with increasing $\rho_N$ in the bottom half of the pedestal primarily because the analytic formula for the ETG-induced transport is inversely proportional to the electron density $n_e$ in this collisional regime. But in the bottom half (III) of the pedestal ETG-induced transport might be limited by the gyro-Bohm scaling factor indicated in Fig. 11 and hence be negligible there. Direct gyrokinetic simulations (for example, as in [34, 44]) of ETG- and TEM-induced microturbulence, the electron heat transport they induce, and the $T_e$ profile they produce are needed to obtain more precise theoretical predictions for these pedestal transport processes.

In summary, none of these electron heat transport models can be unequivocally ruled in or out by this comparison of simple analytic-based theoretical models with the interpretive, effective electron heat diffusivities in this 98889 edge region. However, the paleoclassical model provides perhaps the best overall fit in the pedestal regions where it can be appropriately determined. Also, ETG transport is likely to be a major contributor in the transition from the pedestal (II) into the core (I) region.

Figure 13 shows that for ions the analytic-based theoretical predictions also differ some from the interpretive, effective ion heat diffusivity. As in the discussion of Fig. 11, the neoclassical ion heat diffusivity is larger than the interpretive $\chi_i$ by a factor of about three at $\rho_N \sim 0.965$ and has the wrong profile. Here, the neoclassical $\chi_i$ has been evaluated using the Chang-Hinton formula [39] specified in Eq. (1) of Ref. [43] using the reduction factor given in Eq. (6) of Ref. [43] to account for orbit squeezing [42].

The ITG transport seems too high in the core region going into the top half (II) of the pedestal; it is not present in the lower parts of the pedestal because the ITG instability criterion in Eq. (7) of Ref. [43] is not satisfied, mainly because $\eta_i < 1.25$ there. However, since the plasma is obviously well above ITG-threshold conditions in the core going into the top half of the pedestal, full gyrokinetic turbulence simulations [44] are needed to clarify the magnitude of the ion heat flux in the core region of this 98889 discharge. Drift-Alfvén microturbulence-induced ion heat transport is apparently negligible in the core but could contribute in the bottom half (III) of the pedestal.

In summary, for ion heat transport, ITG microturbulence-induced transport likely plays a dominant role in the core region (I). Neoclassical transport is likely to be a major contributor in the pedestal region (II and III), where it even seems to be too large by perhaps a factor of three. However, more detailed gyrokinetic-based simulations will be required to clarify the precise roles and magnitudes of ITG-induced, and perhaps other ion heat pedestal transport processes, including possible ion heat pinch effects.
6. Predictive 1.5D modeling of pedestal electron temperature profile

The preceding transport analyses have focused primarily on interpretive modeling of the 98889 pedestal. In this section for completeness we briefly discuss some ASTRA $T_e$ predictive modeling results [45]. In predictive transport modeling one uses formulas from various analytic-based theoretical models for diffusivities or transport fluxes as input and simulates the plasma transport evolution to steady state. It seeks to match, within some statistical error, the experimentally obtained quasi-equilibrium plasma profiles.

Previous predictive transport modeling of H-mode plasmas have usually had difficulty modeling the electron temperature profiles — mainly because they did not have a good model of the electron heat diffusivity $\chi_e$ in the pedestal. However, recent ASTRA modeling [45] has been quite successful in modeling the overall $T_e$ profiles in 15 DIII-D H-mode discharges. Two aspects of the modeling are critical. First, the density and ion temperature profiles were held fixed; thus, only the electron temperature profile was modeled and evolved to steady-state. Second, a wide variety of analytic-based theoretical models were considered and a “particular combination of electron thermal transport models” was developed and applied to all 15 DIII-D H-mode discharges.

The particular electron heat transport model that worked best was comprised of ETG-induced and paleoclassical transport models. In particular, the electromagnetic limit of the Horton ETG model [46] was used. In addition, the GLF23 electron heat transport model [47] with the TEM component suppressed (presumably due to $E \times B$ flow shear) was used; thus, only GLF23’s electrostatic version of ETG-induced transport is retained. Finally, a paleoclassical electron heat transport model [48] was included.

The contributions of these electron heat transport models to the ASTRA modeling of the 98889 edge region are shown in Fig. 14 for the 3962 ms time slice. These predictive ASTRA diffusivities are somewhat larger in magnitude but have the same profile as the interpretive results in Fig. 8a. These results show that in the core (I) all three models contribute about equally. In the top half (II) of the pedestal ETG transport becomes small and paleoclassical transport begins to become dominant. Finally, in the bottom half (III) of the pedestal the paleoclassical electron heat transport completely dominates.

The electron temperature profile produced by this combination of electron heat transport models is shown in Fig. 15. The ASTRA modeled and experimental $T_e$ profiles are in reasonable agreement. However, the ASTRA-modeled electron temperature in the pedestal (i.e., in regions II and III) is somewhat larger than the $T_e$ experimental profile there. In part, this could be the result of only using the lowest order paleoclassical electron heat transport model [48]; heat-pincho-type effects that result from the different structure [49] of the paleoclassical electron heat transport operator could reduce the effective $\chi_e^{\text{Paleo}}$ in the pedestal region by a factor of two or more and thereby increase the magnitude of the electron temperature gradient in the top half of the pedestal.
7. Interpretive 2D modeling

There are two key issues for 1.5D transport modeling in the pedestal region. They are: 1) How far out in radius is it appropriate to use the 1.5D transport model?; and 2) what 2D physics effects need to be introduced to extend the transport modeling out to the separatrix ($\rho_N = 1$)? These questions will be addressed in this section with 2D transport modeling, primarily using SOLPS results [50].

These SOLPS 2D transport studies are semi-predictive in that they use the interpretive analysis diffusivities from ONETWO as input and then adjust them slightly in a predictive mode to obtain good agreement with the experimentally measured profiles shown in Fig. 3. This two-step, semi-predictive procedure provides [50] a very efficient new approach for obtaining transport results with the SOLPS modeling code.

First, we consider the poloidal variation of the electron and ion temperatures on magnetic flux surfaces. Relevant results from SOLPS 2D modeling of the pedestal are shown in Figs. 16, 17. For reference, the mapping of $\rho_N$ flux surfaces to $R_{\text{sep}} - R$, which is the radial distance in from the separatrix at the vertical elevation of the magnetic axis, is indicated in Table I. As shown in Figs. 16 and 17, the poloidal temperature variations on flux surfaces throughout the pedestal region are negligibly small (compared to the variation in the magnetic field strength, $\epsilon \sim 0.34$). Even at the outermost surface shown...
Analysis of pedestal plasma transport

Figure 15. ASTRA predictive $T_e$ modeling results [45] for the combination of electron heat diffusivities shown in Fig. 14 produces reasonable agreement with the experimentally measured profile in the 98889 edge region.

Table 1. Mapping of radial distance (in m and approximate cm) between flux surfaces and the separatrix (on outboard mid-plane) to the normalized radial coordinate $\rho_N$. 

<table>
<thead>
<tr>
<th>$R - R_{sep}$</th>
<th>$R_{sep} - R$</th>
<th>$\rho_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.000886 m</td>
<td>0.09 cm</td>
<td>0.99662</td>
</tr>
<tr>
<td>-0.00268 m</td>
<td>0.27 cm</td>
<td>0.99062</td>
</tr>
<tr>
<td>-0.00451 m</td>
<td>0.45 cm</td>
<td>0.98455</td>
</tr>
<tr>
<td>-0.00554 m</td>
<td>0.55 cm</td>
<td>0.98</td>
</tr>
<tr>
<td>-0.00640 m</td>
<td>0.64 cm</td>
<td>0.97832</td>
</tr>
<tr>
<td>-0.01030 m</td>
<td>1.03 cm</td>
<td>0.96564</td>
</tr>
<tr>
<td>-0.01670 m</td>
<td>1.67 cm</td>
<td>0.94528</td>
</tr>
<tr>
<td>-0.0144 m</td>
<td>1.44 cm</td>
<td>0.95</td>
</tr>
<tr>
<td>-0.0175 m</td>
<td>1.75 cm</td>
<td>0.94</td>
</tr>
<tr>
<td>-0.0307 m</td>
<td>3.07 cm</td>
<td>0.90</td>
</tr>
<tr>
<td>-0.0487 m</td>
<td>4.87 cm</td>
<td>0.85</td>
</tr>
<tr>
<td>-0.0682 m</td>
<td>6.82 cm</td>
<td>0.80</td>
</tr>
</tbody>
</table>
(about 1 mm inside the separatrix, at $\rho_N \simeq 0.997$), the electron and ion temperatures vary by only about 0.13 % and 6.5 %, respectively. In the SOLPS transport model the “source” that induces these small, approximately sinusoidal poloidal variations in the temperatures is the peaking of the radial heat fluxes on the outboard mid-plane shown in Fig. 6. The magnitude of the induced poloidal asymmetry is limited by the dissipative relaxation caused by the very large collisional [11] parallel heat conduction. The poloidal variation in the ion temperature is about a factor $\sim 50$ larger than that in the electron temperature because parallel ion heat conduction is smaller than electron heat conduction by a factor $\sim (m_D/m_e)^{1/2} \sim 60$. Since these temperature variations within flux surfaces are so small, the assumption in the 1.5D modeling codes, which was used in the preceding section, that temperatures are approximately constant on flux surfaces is well justified on all flux surfaces inside the separatrix.

The 2D behavior of the density is more complicated — because the recycling of thermal neutrals that provides the plasma fueling in the edge comes predominantly from the poloidally localized X-point region of the LSN divertor separatrix in 98889 (see Fig. 2 and Ref. [50]). A typical contour plot of the 2D neutral density distribution from UEDGE run in an interpretive mode is shown in Fig. 18. It clearly shows that the neutral density is very strongly peaked near the X-point region. Figure 19 illustrates the Monte-Carlo-based SOLPS modeling of the very large poloidal variation of the
deuterium ion source rate $S_n$ just inside the separatrix induced by thermal neutral fueling from recycling in the divertor region. Since, as shown in Fig. 2, the plasma is relatively far from the upper baffles and outboard wall, neutral recycling from the chamber walls has been neglected in this SOLPS modeling [50]. In the pedestal the ion source is a factor of the order of $10^2$ smaller at the inboard, outboard mid-planes and top of the flux surface compared to the ion fueling rate near the X-point. Neutral recycling from the chamber walls could add to the thermal ion source in the top half of the plasma and thereby reduce the asymmetry factor somewhat. However, this should not significantly change the flux-surface-average recycling ion source rate obtained from these Monte-Carlo-based SOLPS modeling results, which is shown below in Fig. 22.

The very strong poloidal asymmetries in the recycling neutral density in Fig. 18 and consequent ion source $S_n$ in Fig. 19 together with the poloidal variation of the radial particle flux (similar to Fig. 6) yield the poloidal variations of the electron and deuterium and carbon ion densities shown in Fig. 20. The poloidal variation in $n_e$ is so small (at most of order 4 %, even just inside the separatrix) because of the large electrical conductivity and consequent equilibration of electron density along magnetic field lines. This small electron density variation implies a Boltzmann-type potential variation of $e \delta \Phi / T_e \sim \delta n_e / n_{e0} \sim 0.04$. Electron density and potential variations within flux surfaces of about these magnitudes have been experimentally measured or inferred
Figure 18. 2D neutral density distribution obtained from a typical UEDGE modeling of 98889 shows recycling neutrals are strongly peaked in the LSN X-point region.

previously (see Fig. 6 of Ref. [51]) at and just inside the separatrix in DIII-D low power H-mode LSN discharges. The poloidal variation in plasma density in Fig. 20 is very small and much less than the fractional variation of the magnetic field strength (\( \sim \epsilon \sim 0.34 \)). Thus, the assumption in the 1.5D modeling codes, which was used in the preceding section, that the electron density is approximately constant on flux surfaces in the edge plasma is well justified on all flux surfaces inside the separatrix.

The deuterium ion density shown in Fig. 20 is peaked in the X-point region because the dominant divertor recycling source is strongly peaked there. It is less than its FSA on the outer mid-plane because radial particle flux losses are largest there for the Fick’s diffusion law model in (10) used in the SOLPS modeling of this discharge [50]. The carbon densities (of C\(^{6+}\) and C\(^{5+}\)) are peaked near the outer mid-plane because, as for the electron and ion temperature variations in Figs. 16 and 17, their primary “source” is provided by radial transport outward from the core plasma, which is peaked near the outer mid-plane (see Fig. 6). More details about SOLPS 2D modeling of deuterium and carbon ion densities and flows and their effects on this discharge can be found in [50].

In summary, despite the very large poloidal asymmetry in the recycling neutral
density in Fig. 18 and the consequent ion recycling source shown in Fig. 19, it appears that on all flux surfaces inside the separatrix the electron density is essentially constant on flux surfaces. And recall that the electron temperature is also essentially constant on flux surfaces in the pedestal. Hence the net particle and electron energy transport in the pedestal can clearly be described by the 1.5D equations in (6) and (7).

As shown in Fig. 20, the ion and carbon densities vary poloidally by factors of about two just inside the separatrix in this SOLPS modeling. However, as shown in Fig. 21, the poloidal variation of the deuterium ion density about its FSA rapidly becomes very small on successive flux surfaces inside the separatrix. In particular, it drops to less than about 15 % by the next modeled flux surface at $R_{\text{sep}} - R = 0.00268$ m. Thus, except very close to the separatrix (perhaps for $\rho_N > 0.99$, $R_{\text{sep}} - R < 2.6$ mm), the deuterium and carbon ion densities can also be assumed to be approximately constant on flux surfaces. Deuterium ion orbit losses in this region could introduce “kinetic” poloidal asymmetry effects near the separatrix. However, presumably only a small fraction of energetic “tail” ions would be on lost orbits because thermal ions in this region (III) are in the transition from the banana to plateau collisionality regime ($\nu_{si} \gtrsim 0.5$).
Hence, the effects of the 2D particle and energy sources will apparently enter transport analyses predominantly through their flux surface averages, at least out to about $\rho_N > 0.99$. Therefore, within the 1.5D pedestal transport modeling paradigm, the role of the 2D modeling codes is to determine the 2D character of the sources and sinks in the pedestal induced by the 2D effects originating outside the separatrix. Then the flux surface averages of them are included on the right hand sides of the transport equations (6) and (7).

The flux surface average of the modeled recycling ion source is shown in Fig. 22. This source $\langle S_n \rangle$ is given by the FSA of the product of the neutral density and the $n_e \langle \sigma v \rangle$ collision rate for ionization; i.e., it is proportional to the neutral particle density. The ONETWO results in Fig. 22 are obtained using a 1D cylindrical model for the recycling neutrals [52] modified to take account of the “flux expansion” effects near the divertor via the Mahdavi model [53]. The 2D SOLPS neutral model is a kinetic Monte-Carlo-based model whose recycling ion source results are shown in Fig. 19. Its $\langle S_n \rangle$ is larger than the ONETWO value at the separatrix because in SOLPS the diffusion coefficient at the separatrix $D(R_{\text{sep}})$ was increased about a factor of two to facilitate matching the density profile in the pedestal [50]. Its $\langle S_n \rangle$ is below the ONETWO values.
for $R_{\text{sep}} - R \gtrsim 6.5$ cm ($\rho_N \lesssim 0.8$) because this SOLPS modeling does not include the core neutral beam ionization source whereas ONETWO does. The UEDGE code uses a 2D fluid-based neutral transport model [9]. The GTEDGE code obtains the 2D neutral density distribution from the GTNEUT code [54, 55] which uses an integral transport calculation for the neutral transport based on Transmission-Escape-Probability (TEP) theory. The variability in the recycling plasma source from the divertor region is indicative of the largest uncertainty in the present pedestal transport analyses. More divertor data is needed [50] to reduce these uncertainties.

8. Density transport

As a first step in exploring density transport, in Fig. 23 we plot the net flow rate $\dot{N}$ (s$^{-1}$) defined in (8) of charged particles through flux surfaces as a function of $\rho_N$. It is obtained from ONETWO, SOLPS and UEDGE modeling of the edge plasma we are considering. In the core region (I) the particle flow rate is approximately constant at $\sim 0.44 \times 10^{21}$ s$^{-1}$ or 70 Amperes equivalent. Its near constancy indicates the neutral beam fueling of the core plasma and its nearly uniform radial transport outward. The rapid increase of $\dot{N}$ in the pedestal region (II and III) is caused by recycling neutrals providing the edge ion source shown in Fig. 22. The net increase in outward particle flow $\dot{N}$ over the pedestal of (from ONETWO and UEDGE) $\sim 7 \times 10^{20}$ s$^{-1}$ ($\sim 110$ Amperes equivalent)
Figure 22. Flux surface average of the recycling source of ions obtained from ONETWO and SOLPS [50] modeling is concentrated near the separatrix.

Figure 23. Particle flow rate $\dot{N}$ ($\times 10^{21} \text{ s}^{-1}$) increases with $\rho_N$ in the pedestal region.
Figure 24. Effective density diffusivity $D$ (m$^2$/s) versus $\rho_N$ obtained from GTEDGE, ONETWO, SOLPS and UEDGE interpretive modeling is very small in the pedestal.

is approximately given by $\delta \dot{N} \simeq \langle S_n \rangle \delta V \simeq (5 \times 10^{20} \text{ m}^{-3}\text{s}^{-1}) \cdot 1.5 \text{ m}^3 = 7.5 \times 10^{20} \text{ s}^{-1}$. All these modeling codes indicate that the edge recycling fuels the pedestal region (II and III) but is mostly negligible from the top of the pedestal inward into the core (I).

The interpretive, effective density diffusivity $D$ is obtained by dividing the net particle flow through a flux surface by the density gradient, as defined in (12). The interpretive $D$ profiles obtained this way from the modeling codes are shown in Fig. 24. There are a number of interesting features of these results. First, it is encouraging that the modeling codes obtain much the same effective density diffusivity profiles, despite significant differences in their analysis procedures and ion source rates, as shown in Figs. 22 and 23. Next, note that the shapes of the curves are similar to those for effective heat diffusivities in Fig. 8. Analogous to the reasons for the shapes of the $\chi$ profiles, as $\rho_N$ increases outward from the core (I) and down the top of the pedestal (II), the interpretive $D$ decreases mainly because the magnitude of the density gradient is increasing. Then, after passing the mid-point of the pedestal (III), the effective $D$ increases with increasing $\rho_N$ because of both the increasing particle flow rate $\dot{N}$ and the decreasing magnitude of the density gradient there. It is also noteworthy that the minimum density "transport barrier" value of the effective $D$ is very small. It is about 0.035 m$^2$/s (± a factor of almost two), which is almost an order of magnitude smaller than the effective heat transport barriers in Fig. 8.

In tokamak plasmas it is often inferred [56] that density transport in the core plasma is not purely diffusive. Instead, the radial density flux seems to be a combination of
diffusive outward density transport as in (10) balanced by an inward density pinch flow:

\[ \Gamma = - D \nabla n + n \mathbf{V}_{\text{pinch}}. \]  

(14)

Here, \( \mathbf{V}_{\text{pinch}} \) is a negative velocity (m/s) which indicates an inward "pinch" flow velocity of the species density. Note that \( \mathbf{V}_{\text{pinch}} \) represents radial flow of the species particle distribution as a whole. Thus, it does not necessarily imply that individual particles have an inward "pinch velocity" component; rather, it could just be a fluid moment property of a given plasma species in a tokamak. Alternatively, it could represent: "off-diagonal" transport fluxes (e.g., due to temperature gradients), threshold density gradients (e.g., due to KBMs [26]) beyond which fluctuation-induced transport dramatically increases to produce a form of local "profile resiliency," or more generally that, as in the paleoclassical density transport model (see Eqs. (74), (75) and (78) in [57]), the density transport operator does not result from a Fick’s diffusion law particle flux as in (10).

There are a number of indications that a strong pinch effect is operative in H-mode pedestals. It was shown in 1993 that “Flux-surface-averaged transport modeling of the time evolution for the core plasma density profile during H mode suggests that a strong inward particle pinch is necessary near the separatrix” [58]. Here, we note that at the top of the pedestal (i.e., at \( \rho_N \approx 0.94, R_{\text{sep}} - R \approx 1.75 \) cm) where the density is \( \approx 3 \times 10^{19} \) m\(^{-3}\), the ion source rate from Fig. 22 is \( \langle S_n \rangle \approx 2 \times 10^{20} \) m\(^{-3}\) s\(^{-1}\). Thus, at the top of the pedestal (boundary between I and II) the time scale for all this density to re-build via \( \langle S_n \rangle \) after an ELM is \( \tau_n \approx n_e / \langle S_n \rangle \approx 150 \) ms. However, this time is about an order of magnitude longer than the \( \tau_n \approx 10 \) ms electron density re-build time observed in the 98889 pedestal (see bottom waveform in Fig. 1 and discussion of it). The pinch velocity required to re-build the density at the top of the pedestal on this time scale is modest: \( V_{\text{pinch}} \approx (1 - \rho_N) a / \tau_n \approx 0.045 \) m/0.01 s \( \approx 4.5 \) m/s. Other indications that pinch effects might be operative in the core plasma just inside the top of the pedestal are: 1) the interpretation of ELM-induced transient responses of helium impurities in DIII-D H-mode plasmas imply an inward pinch velocity for them at \( \rho_N \approx 0.65 \) of about 1 m/s which increases with \( \rho_N \) [59]; and 2) the often-observed “ears” [58, 60, 61] on radial profiles of electron density that peak just inside the top of the density pedestal apparently require a pinch-type effect to produce them.

9. Inferring, explaining the density pinch and its consequences

Up to now there has been no way to determine the pinch velocity experimentally, except by analyzing responses to spatially localized transients [56]. However, a recent series of papers [62] have developed an important new interpretive procedure for inferring the pinch velocity. It is based on using [28, 62], in addition to the usual density and energy transport equations, the momentum conservation (force balance) equations. The full procedure is rather complicated [62]. In the following we provide a brief description of its procedural logic and an alternate justification for its key assumption. Thereafter, we present the results obtained from this new procedure and compare them to predictions of the paleoclassical density transport model.
The new procedure can be explained in terms of a multiple-time-scale approach [57, 63] to the plasma force balance equations, as follows. On time scales longer than the fast, compressional Alfvén wave time scale ($\sim \mu$s), the radial force balance yields a (cylindrical-type) relation between the toroidal flow $V_{\phi}$, poloidal flow $V_{\theta}$, and $E \times B$ and diamagnetic flows for the $j$th species of plasma particles:

$$V_{\phi j} \simeq \frac{E_r}{B_\theta} - \frac{1}{n_j q_j B_\theta} \frac{dp_j}{dr} + \frac{B_\phi}{B_\theta} V_{\theta j}.$$  \hspace{1cm} (15)

Here, $B_\phi$ and $B_\theta$ are the magnitudes of the toroidal and poloidal components of the magnetic field. The more general flux coordinate form of (15) is given in Eq. (8) of [63].

Next the component of the force balance equations along the magnetic field $B$ is considered [63]; because of the axisymmetry, this is equivalent to the poloidal force balances used in [62]. The parallel/poloidal force balances relax the poloidal flows to their equilibrium values via neoclassical parallel viscous forces [64] on the ion collision time scale, which is of order 1 ms in the pedestal region (II and III). Details of the determination of the equilibrium poloidal flows are given in [62, 63, 65]. The neoclassical-determined poloidal flows in the pedestal are small contributors to (15) and will mostly be neglected in the following discussion.

Finally, the cylindrical form of the toroidal force balance equation yields a transport time scale equation for the toroidal flow velocity of an ion species $j$ (deuterons or carbon in 98889):

$$m_j n_j \frac{dV_{\phi j}}{dt} = e_j \Gamma_{rj} B_\theta + \sum_{k} T_{\phi jk}/R + \sum_{l} M_l.$$  \hspace{1cm} (16)

Here, the radial particle flux of the $j$th species is $\Gamma_{rj}$, the sum over $k$ is over the various collisional and microturbulence-induced toroidal torques $T_{\phi jk}$ on the $j$th ion species, $R$ is the major radius, and the sum over $l$ is over the externally applied momentum inputs $M_l$ (e.g., due to energetic neutral beams), which are negligible in the pedestal. Analogous general flux coordinate forms of the equation for the FSA toroidal rotation frequency $\Omega_t \equiv \mathbf{V} \cdot \nabla \zeta \simeq V_\phi/R$ are given in Eq. (119) in Ref. [57] and Eq. (22) in Ref. [63].

In equilibrium, in the absence of any significant toroidal momentum sources, this last equation yields

$$\Gamma_{rj} = -\sum_{k} \frac{T_{\phi jk}}{e_j n_j q_j B_\theta}.$$  \hspace{1cm} (17)

This relation shows that the radial particle (density) flux is caused by the sum of the toroidal torques on the plasma species. The 7 intrinsically ambipolar particle flux components induced by collisional torques and the 8 non-ambipolar particle flux components induced by microturbulence, 3D non-axisymmetric (NA) magnetic field components, and other effects are discussed in Ref. [57].

The key assumption in the new interpretive procedure for determining the radial pinch velocity [62] is that the "anomalous" toroidal torques (due to microturbulence, 3D field components etc.) on a plasma species $j$ can be written as

$$T_{\phi j}^{\text{an}} / R \equiv -m_j n_j \nu_\phi V_{\phi j}.$$  \hspace{1cm} (18)
Here, $\nu_{dj}$ is a “drag” frequency on the toroidal flow caused by the various anomalous plasma transport processes. It is determined [62, 66] from poloidal and toroidal carbon flow experimental data using a combination of the relations in (15) and (16) for the deuteron and carbon species in the edge plasma.

Once the collisional drag frequency $\nu_{dj}$ is determined, substituting (18) into (17) and using (15), the radial ion flux of the species $j$ is given by the “pinch-diffusion” relation [62, 66]

$$\Gamma_{rj} \simeq -D_j \left( \frac{1}{T_j} \frac{dp_j}{dr} \right) + n_j V_{rj}^{\text{pinch}}. \quad (19)$$

Here, the density (pressure) diffusivity and radial pinch velocity are defined by

$$D_j \simeq \frac{m_j T_j \nu_{dj} e_j^2}{e_j B_\theta^2}, \quad V_{rj}^{\text{pinch}} \simeq \frac{m_j \nu_{dj} e_j B_\theta^2}{e_j B_\theta^2} (E_r + V_{\theta j} B_\phi). \quad (20)$$

These formulas are only approximate, illustrative versions of the more comprehensive equations that have been developed [62] and are used for determining these quantities in the 98889 edge plasma [66].

The key assumption that the toroidal torque $T_{\phi j}^{\text{an}}$ can be written in the form given by (18) can be motivated by first observing [63] that the thermodynamic force $\partial f_{Mj}/\partial r$ for collisional and anomalous radial transport processes can be written in the form

$$\frac{\partial f_{Mj}}{\partial r} = f_{Mj} \left[ \frac{1}{p_j} \frac{dp_j}{dr} + \frac{e_j}{T_j} \frac{d\Phi}{dr} + \left( \frac{E}{T_j} - \frac{5}{2} \right) \frac{1}{T_j} \frac{dT_j}{dr} \right]. \quad (21)$$

Next, we note that the ion temperature gradient in the pedestal region (II and III) is typically somewhat weaker than the electron density and temperature gradients there (see Figs. 3 and 9). Also, for radial transport effects that are not too strongly dependent on the particle energy $E$, the ion temperature gradient term will not contribute significantly when this thermodynamic force is averaged over velocity space. These effects cause the ion temperature gradient term in (21) to not contribute significantly in the pedestal (II and III). They also cause the neoclassical-determined poloidal flows in the pedestal to be small.

Thus, neglecting the poloidal flows in (15) and the ion temperature gradient term in (21), we see that in the pedestal region

$$\frac{\partial f_{Mj}}{\partial r} \simeq -\frac{e_j B_\theta f_{Mj}}{T_j} V_{\phi j}. \quad (22)$$

Hence, the thermodynamic force for radial plasma transport is roughly proportional to the toroidal flow velocity $V_{\phi j}$ [63]. Then, characterizing diffusive anomalous radial transport processes by a microscopic, kinetic diffusion coefficient $(\Delta x)^2/(2\Delta t)$, in which $\Delta x$ is the random radial step taken in a time $\Delta t$ (due to 3D NA collisional effects or plasma microturbulence), the radial ion flux can be written phenomenologically as

$$\Gamma_{rj} \simeq -\int d^3\nu \frac{(\Delta x)^2}{2\Delta t} \frac{\partial f_{Mj}}{\partial r} \simeq D_j \frac{n_j e_j B_\theta}{T_j} V_{\phi j} \equiv \frac{m_j n_j}{e_j B_\theta} \nu_{\phi j} V_{\phi j} = -\frac{D_j}{T_j} \left[ \frac{dp_j}{dr} - n_j e_j E_r \right]. \quad (23)$$
The “neoclassical toroidal viscosity” (NTV) collisional effects of 3D non-axisymmetric toroidal magnetic fields are naturally written in this form when ion temperature gradient effects are neglected — see Eqs. (106) and (100), (101) in Ref. [57]. Microturbulence-induced radial transport of toroidal flow may be diffusive with a coefficient $\chi_\phi$ via the Reynolds stress it induces (see Eqs. (109) and (114) in Ref. [57]). Thus, since the thermodynamic force in (22) is proportional to the toroidal flow $V_{\phi j}$, the confinement time for toroidal momentum in the pedestal is $\tau_\phi \sim (\delta \rho)^2 / \chi_\phi$ in which $\delta \rho$ is the radial width of the pedestal. Therefore, in steady state, microturbulence induces an effective toroidal drag frequency $\nu_{dj} \simeq 1 / \tau_\phi \sim \chi_\phi / (\delta \rho)^2$. Both these examples show that forms like the relations in (23) may result from kinetic-based treatments using the thermodynamic force in (22).

Note that the form of the ion flux obtained in (23) is the same as that proposed in (19) for the ion density (pressure) diffusivity $D_j$ and pinch velocity $V_{\text{pinch}}^{\text{pinch}}$ definitions in (20) when $E_r \gg V_{\phi j} B_\phi$. Thus, when poloidal flows are negligible, consideration of the appropriate thermodynamic force for radial ion transport provides a motivation and possible justification for the new interpretive procedure’s key assumption stated in (18).

The pinch velocity obtained via the new interpretive procedure for the 98889 edge plasma is shown in Fig. 25. In the pedestal the inferred ion pinch velocity is inward and very large — about 100 m/s at the pedestal mid-point (boundary between regions
II and III). Figure 25 also shows that throughout the pedestal region (II and III) the inferred radial pinch velocity is opposite to and about an order of magnitude larger than the interpretive net ion radial ion flow velocity $V_r \equiv \Gamma_{ri}/n_i$. Thus, it seems that the pinch velocity cancels about 90% of the diffusive outward ion flux in the 98889 pedestal in yielding the net radial ion flow velocity $V_r$. However, it should be emphasized that the density pinch discussed here is the result of an ion force-balance-based interpretive procedure and not a direct experimental measurement of it.

The interpretive effective density diffusivity $D$ in the pedestal exhibits an apparent very strong transport barrier near the mid-point of the pedestal (i.e., near $\rho_N \sim 0.98$, between regions II and III) — see Figs. 24 and 26. However, as shown in Fig. 26, taking account of the very strong pinch flow determined by the new interpretive procedure, the inferred (true?) density diffusivity $D_{\text{exp}}$ increases monotonically with $\rho_N$ throughout the pedestal and does not attain a minimum value there. (The $D_{\text{exp}}$ value is unusually small in the core region I in this GTEDGE modeling because the “pinch flow” inferred there is outward not inward — because the neutral beam fueling source has been neglected and perhaps additionally because the poloidal flows become significant there.)

Thus, the very small interpretive effective $D$ near the mid-point of the pedestal may not represent a transport barrier. Rather, it may be an artifact of neglecting the pinch flow in inferring the density diffusivity $D$. These new pedestal results indicate a

---

**Figure 26.** GTEDGE-determined density diffusivities: usual interpretive effective one ($D$, red circles), inferred (true?) one corrected for inward pinch flow ($D_{\text{exp}}$, black squares) and paleoclassical model ($D_{\text{paleo}}$, blue triangles).
diffusively dominant density transport model is likely not appropriate in the pedestal. This has very important and profound implications for the development of a fundamental understanding of radial density transport processes in and fueling of H-mode pedestals.

Unlike the situation with radial electron and ion heat transport, there are very few analytic-based theoretical models for particle diffusivities, pinch flow velocities, and net density transport fluxes, beyond the negligible neoclassical predictions. This is especially true for the plasma parameters in H-mode pedestals. Recently, the paleoclassical transport process [36, 37] has been identified (see discussion after Eq. (125) in [57]), after the toroidal rotation and hence ambipolar radial electric field are determined and used, as a likely dominant net ambipolar density transport process. As indicated in Eqs. (78) and (125) in [57], the paleoclassical radial density transport flux is

$$\Gamma_{\text{r paleo}} = -\frac{1}{V'} \frac{\partial}{\partial \rho} (V' \bar{D}_{\eta} n) = -\bar{D}_{\eta} \frac{\partial n}{\partial \rho} + n V_{\text{pinch}}^{\text{paleo}}. \quad (24)$$

The paleoclassical density diffusivity $\bar{D}_{\eta}$ and pinch flow velocity $V_{\text{pinch}}^{\text{paleo}}$ are

$$\bar{D}_{\eta} \equiv D_{\eta} \left\langle \frac{(|\nabla \rho|^2 / R^2)}{\langle R^{-2} \rangle} \right\rangle, \quad V_{\text{pinch}}^{\text{paleo}} \equiv -\frac{1}{V'} \frac{d}{d\rho} (V' \bar{D}_{\eta}^{\text{paleo}}), \quad (25)$$

in which $D_{\eta} \equiv \eta_{\parallel}^{\text{nc}} / \mu_0 \sim 10^3 \, Z_{\text{eff}} / T_e (\text{eV})^{3/2} \, \text{m}^2 / \text{s}$ is the magnetic field diffusivity induced by the parallel neoclassical resistivity $\eta_{\parallel}^{\text{nc}}$ (see blue dash-dot $D_{\eta}$ curve in Fig. 11) and as above $V' \equiv dV / d\rho$ is the radial derivative of the flux surface volume.

Approximate predictions of the paleoclassical transport model for the density pinch flow and diffusivity are indicated by the blue triangle curves in Figs. 25 and 26. Here, for simplicity, the radial variations of $V'$ have been neglected and in $\bar{D}_{\eta}$ the FSA geometry factor (it is somewhat smaller than the $\left\langle |\nabla \rho|^2 \right\rangle$ factor shown in Fig. 7) has been set to unity; thus, in Fig. 25 $V_{\text{pinch}}^{\text{paleo}} \sim -dD_{\eta} / d\rho$ and in Fig. 26 $D_{\text{paleo}} \sim D_{\eta}$. As can be seen, in the pedestal the paleoclassical predictions are within about a factor of two of both the inferred pinch velocity and density diffusivity determined from the new interpretive procedure. Therefore, the paleoclassical transport model provides a plausible explanation of these important new interpretive results.

10. Discussion: what is needed to progress?

The preceding analysis of pedestal transport has highlighted many areas where present modeling is not complete or is inconclusive. In this section we discuss some areas where additional research is needed and some possible new research directions. The various areas discussed in the following paragraphs are: heat transport modeling, density transport, new DIII-D pedestals to analyze, and pedestals from other tokamaks.

Heat transport modeling. For electron heat transport, the main needs are for detailed gyrokinetic simulations (e.g., as in [34], [44]) of primarily ETG-induced microturbulence throughout H-mode pedestals, and in particular in their bottom halves (III). Also, the complete paleoclassical electron heat transport operator [48, 49] with its heat-pincho-type effects needs to be taken into account. For ions, a number of
Analysis of pedestal plasma transport

Areas need to be examined in greater detail: precise neoclassical ion heat diffusivity calculations (e.g., as in [40]), kinetic calculations including the effects of electric-field-induced orbit squeezing and ion orbit losses, ITG-induced microturbulence simulations, and consideration of possible paleoclassical ion heat diffusivity and pinch effects. Also, for KBMs [26] precise instability criteria, fluctuation spectra and their nonlinear and transport consequences need to be explored.

Density modeling. This is a crucial area that needs considerably more attention. The general need here is for more recent DIII-D H-mode discharges in which more density-related quantities are measured, particularly in the divertor region. Divertor region measurements are needed to pin down the neutral density and recycling ion source in this region to facilitate a more precise determination of the “pedestal boundary conditions” for these quantities and the radial density transport in the vicinity of the separatrix. Also, direct experimental measurements of density pinches via analysis of transient responses (after L-H transition, ELMs) are needed to experimentally validate the new strongly-coupled density pinch-diffusion model discussed in Section 9. In addition, much more consideration and modeling, including of the dynamics, of density transport with strong pinch effects is needed. Further, the degree of importance of 2D effects may be able to be explored by determining the poloidal variation of the carbon density just inside the separatrix (see Fig. 20). Finally, poloidal and toroidal plasma flows in the vicinity of the separatrix need to be explored [50] to determine the possible roles and importance of such effects in pedestal plasma transport.

New DIII-D pedestals to analyze. In general, more pedestals from DIII-D need to be analyzed to see how universal the pedestal transport properties identified in this paper are. Particularly useful would be pedestals with the following characteristics: 1) better diagnosed pedestals with more divertor region data; 2) pedestals with higher and lower collisionality; 3) wider pedestals with more spatial resolution, particularly in the bottom half of pedestal; and 4) microturbulence fluctuation data throughout the pedestal looking in particular for intermittent bursts of fluctuations (e.g., due to KBMs [26]) just before ELMs that could presage their occurrence or cause “profile resiliency.” Finally, studies of paired DIII-D discharges with opposite grad-B drift directions would be useful for exploring flows and their effects both outside and just inside the separatrix.

Pedestals from other tokamaks. Of course the ultimate test of one’s understanding is that it also applies to other tokamak plasmas. Thus, it is desirable to analyze H-mode pedestals from other tokamaks with a focus on: well-diagnosed pedestals with long transport quasi-equilibria after an L-H transition and between ELMs; and the widest possible range of machine and dimensionless parameters.

11. Summary

Plasma transport in one DIII-D pedestal (98889) was modeled with many transport modeling codes: 1.5D interpretive (ONETWO, GTEDGE), 1.5D predictive (ASTRA) and 2D (SOLPS, UEDGE) ones. The general conclusions from this multi-faceted study
are: 1) magnetic flux surface geometry effects ($q \propto \nabla \rho$, $\langle |\nabla \rho|^2 \rangle$ factor) are important in the pedestal; 2) the modeling codes generally agree on effective diffusivities if proper comparisons are made; 3) the 1.5D transport models are apparently appropriate for $\rho_N \lesssim 0.99$ using the flux-surface-averaged 2D sources and sinks, with the caveat that kinetic effects due to ion orbit losses from the bottom half of the pedestal could modify this fluid-model-based conclusion for the ions; and 4) the largest uncertainties are in the magnitudes of the recycling thermal neutral density and ion source coming from the divertor X-point region, which influence ion heat and density transport.

Tentative conclusions about plasma transport in the 98889 DIII-D H-mode pedestal are: 1) “transport barrier” interpretive, effective diffusivities are small: 0.3 ($e$ heat), 0.15 ($i$ heat), 0.035 ($n$) m$^2$/s; 2) generic scalings, and interpretive and predictive transport modeling all indicate that electron heat transport is likely dominated by ETG effects at the pedestal top (I to II transition) plus paleoclassical transport throughout the edge plasma region (I, II and III); 3) the less well quantified ion heat transport is likely ITG-induced at the pedestal top (I and transition into II) with a significant neoclassical component in the pedestal (II and III) that may be too large in region II and has the wrong profile there; and 4) density transport may be determined, using a new interpretive procedure, by a strong pinch flow that nearly cancels the outward diffusive density flux, which indicates that the usually inferred deep density transport barrier may be an artifact of neglecting the pinch, and the inferred $V_{\text{pinch}}$ and $D_{\text{exp}}$ are approximately given by the predictions of the paleoclassical density transport model. While there are strong modeling, interpretative analysis and paleoclassical model predictions for a strong density pinch in the pedestal, it should be emphasized that there is as yet no direct experimental measurement of the density pinch and/or its effects.

This study has identified (in Section 10) the following needs for reaching more definitive conclusions about plasma transport in H-mode pedestals: 1) divertor data to better determine the flux-surface-average ion source from the divertor region and the resultant radial density transport on the separatrix; 2) microturbulence simulations and neoclassical calculations for the entire pedestal region; 3) much greater attention to density transport in the pedestal including plausible strong pinch flow effects; and 4) analysis of other DIII-D H-mode pedestals and pedestals from other tokamaks.

Acknowledgments

This paper was prepared for presentation at the 12th International Workshop on “H-mode Physics and Transport Barriers,” 30 September – 2 October 2009, Princeton Plasma Physics Laboratory, Princeton, New Jersey. The authors are grateful to R.E. Waltz for drawing our attention to the fact that the usual ONETWO output files for the interpretive heat diffusivities give $\langle \nabla |\nabla \rho|^2 \rangle \chi$, not $\chi$. This research was supported by U.S. DoE grants and contracts DE-FG02-92ER54139 (UW), DE-FC02-04ER54698 (GA), DE-AC05-00OR22725 (ORNL), DE-FG02-92ER54141 (Lehigh), DE-AC52-07NA27344 (LLNL), DE-FG02-00ER54538 (GaTech).
Appendix A. Coordinate systems used in 1.5D and 2D modeling codes

To specify the divergences of particle and energy fluxes in (1) and (2), we need to specify a coordinate system. Because the various codes use different coordinate systems, we will utilize a generic coordinate system that encompasses all of them. The coordinates of a general curvilinear coordinate system are usually identified as \( u^i \equiv (u^1, u^2, u^3) \). Because of the toroidal axisymmetry, the third coordinate will always be taken to be the toroidal angle: \( u^3 \equiv \zeta = \phi \). Since we are mainly interested in “radial” transport across flux surfaces, it is convenient to choose the radial coordinate to be a flux surface label. To be general we use \( u^1 \equiv x_\psi \), in which \( x_\psi \) is \( \rho, \rho_N, \Psi_N, \) or \( R - R_{sep} \) on the horizontal mid-plane. Finally, a poloidal angle variable is needed. For it we use \( u^2 \equiv x_\theta \), in which \( x_\theta \) is a poloidal angle \( \theta \), or a poloidal distance \( \ell \) (or \( x \) in 2D codes) along a flux surface. The Jacobian of the transformation from laboratory \((x)\) to the (possibly nonorthogonal) \( u^i \) curvilinear coordinates is \( \sqrt{\mathcal{g}} \equiv 1/\nabla u^1 \cdot \nabla u^2 \times \nabla u^3 = (d\psi_p/dx_\psi)/B \cdot \nabla x_\theta \).

The specific coordinates used in the various transport modeling codes are:

ONETWO: \( x_\psi \to \rho \equiv \sqrt{\Phi/\pi B_{10}} \) (m), \( x_\theta \to \ell \) (m), \( \sqrt{\mathcal{g}} \equiv 1/B \cdot \nabla \ell \equiv 1/B_p(\rho, \ell) \).

ASTRA: \( x_\psi \to \rho \equiv \sqrt{\Phi/\pi B_{10}} \) (m), \( x_\theta \to \theta \) (radians), \( \sqrt{\mathcal{g}} \equiv 1/B \cdot \nabla \theta \equiv \rho/B_{pol}(\rho, \theta) \).

GTEDGE: \( x_\psi \to \bar{\ell} \) (m), \( x_\theta \to \theta \) (geometric angle, radians), \( \sqrt{\mathcal{g}} \equiv H = 1/B \cdot \nabla \ell_\theta \).

SOLPS, UEDGE: \( x_\psi \to y \) (m), \( x_\theta \to x \) (m), \( \nabla x_\psi = \hat{e}_y/h_y, \nabla x_\theta = \hat{e}_x/h_x, \sqrt{\mathcal{g}} = h_x h_y R \).

In the ONETWO and ASTRA codes the flux surfaces are determined from the EFIT Grad-Shafranov solver [13]. The GTEDGE code uses [67, 28] the analytic Miller equilibrium [27] model of the EFIT flux surfaces to calculate the poloidal \((x_\theta \to \theta)\) dependence of \( \nabla T \) and \( \nabla n \) (see Section 4); it calculates heat and particle fluxes in an equivalent circular cross-section geometry model that conserves flux surface area in the elliptical approximation, which yields \( \bar{\ell} = r[(1 + \kappa^2)/2]^{1/2} \) in which \( \kappa \) is the vertical elongation of flux surfaces.

In the 1.5D modeling codes (ONETWO, ASTRA, GTEDGE) the radial and poloidal angle coordinates used in developing them are not orthogonal, i.e., \( \nabla x_\psi \cdot \nabla x_\theta \neq 0 \) — because of the toroidal geometry. The nonorthogonal coordinates can be chosen such that, on a flux surface, the magnetic magnetic field lines are straight [i.e., \( \zeta = q(\psi) \theta + \zeta_0 \)]; this is a very useful property for analytic analyses in these low collisionality plasmas where the collision length \( \lambda \) exceeds the poloidal periodicity length \( 2\pi R_0 q \). However on a magnetic separatrix all the 1.5D code coordinate systems become invalid because at the X-point \( B \cdot \nabla x_\theta \) vanishes and the Jacobian is undefined \((\sqrt{\mathcal{g}} = 1/B \cdot \nabla x_\theta \to \infty)\). Thus, these 1.5D code coordinate systems can only be used inside a divertor separatrix.

In the 2D codes (SOLPS, UEDGE) the flux surfaces are taken to be the EFIT equilibrium flux surfaces, which are labeled locally by \( x_\psi \to y \). The poloidal coordinate \( x_\theta \to x \) is defined as being locally orthogonal to the local radial flux surface coordinate. Thus, the 2D \( x, y \) and \( R \zeta \to z \) coordinates represent locally Cartesian coordinates. (It is important to note that this choice of local coordinates is unfortunately the
opposite of that used in the vast plasma instability literature where \( x \) is usually a radial coordinate and \( y \) is a poloidal-type coordinate.) In SOLPS and UEDGE the metric factor \( h_\zeta \equiv 1/|\nabla \zeta| \) for the toroidal direction is taken to be \( 2\pi R \); but for consistency of notation here, we take \( \nabla \zeta = \hat{e}_\zeta/h_\zeta \) with \( h_\zeta = R \) (in m) and integrate over \( \zeta \) to obtain the \( 2\pi \) factor. In the SOLPS, UEDGE formulas above \( \hat{e}_x, \hat{e}_y, \hat{e}_\zeta \) are unit vectors in the \( x, y, \zeta \) directions, respectively. These orthogonal 2D code coordinates are applicable both inside and outside the separatrix and are very useful in the collisional regime outside the separatrix. However, they are not as useful in the hot core plasma because magnetic field lines are not straight in them \([i.e., \zeta = \int_0^x dx' (d\zeta/dx') + \zeta_0 = \int_0^x dx' (B \cdot \nabla \zeta/B \cdot \nabla x) + \zeta_0 = f(x, y) + \zeta_0]\), which makes analytic calculations more difficult with them.

Since for a general curvilinear coordinate system the divergence of a vector \( \mathbf{A} \) is \( \nabla \cdot \mathbf{A} = \sum_i (1/\sqrt{g}) (\partial/\partial u^i) (\sqrt{g} \mathbf{A} \cdot \nabla u^i) \) and because of axisymmetry \( \partial/\partial \zeta = 0 \), we obtain for \( \mathbf{A} = \Gamma \equiv n \mathbf{V} \) [or \( \mathbf{A} \equiv q + (5/2)T \Gamma \)]

\[
\nabla \cdot \Gamma = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x_\theta} (\sqrt{g} \Gamma \cdot \nabla x_\theta) + \frac{1}{\sqrt{g}} \frac{\partial}{\partial x_\psi} (\sqrt{g} \Gamma \cdot \nabla x_\psi),
\]

(A.1)

\[
\sqrt{g} = \nabla x_\psi \cdot \nabla x_\theta \times \nabla \zeta, \quad \text{Jacobian.}
\]

(A.2)

Thus, the density and energy equations can be written in the generic coordinates as

\[
\frac{\partial n}{\partial t} + \frac{1}{\sqrt{g}} \frac{\partial}{\partial x_\theta} (\sqrt{g} \Gamma \cdot \nabla x_\theta) + \frac{1}{\sqrt{g}} \frac{\partial}{\partial x_\psi} (\sqrt{g} \Gamma \cdot \nabla x_\psi) = S_n,
\]

(A.3)

\[
\frac{\partial}{\partial t} \left( \frac{3}{2} nT \right) + \frac{1}{\sqrt{g}} \frac{\partial}{\partial x_\theta} \left( \sqrt{g} (q + \frac{5}{2}T \Gamma) \cdot \nabla x_\theta \right)
+ \frac{1}{\sqrt{g}} \frac{\partial}{\partial x_\psi} \left( \sqrt{g} (q + \frac{5}{2}T \Gamma) \cdot \nabla x_\psi \right) = Q.
\]

(A.4)

These are the equations solved in the 2D modeling codes SOLPS and UEDGE.

In the 2D codes the parallel (to the magnetic field \( B \)) conductive electron and ion heat fluxes are assumed to be given by the classical, collisional Braginskii [11] formulas. The projection of the dominant parallel conductive heat flow in the generic poloidal direction is \( q \cdot \nabla x_\theta = - (B \cdot \nabla x_\theta/B)^2 n \chi_\parallel \partial T/\partial x_\theta \), in which \( \chi_\parallel \) is the classical collisional parallel heat diffusivity. In Section 7 we discuss the 2D code solutions of (A.4); they show that for the pedestal we are considering the very large parallel electron and ion heat diffusivities \( \chi_e \parallel \) and \( \chi_i \parallel \) cause the electron and ion temperatures to become nearly constant along magnetic field lines and hence on flux surfaces throughout the pedestal region. In addition, in the 2D codes the electron parallel collisional friction relaxes the electron density along field lines. Thus, for the present transport analyses the most important plasma transport processes in the pedestal region will be those that transport particles and heat radially across magnetic flux surfaces.
Analysis of pedestal plasma transport

References

Analysis of pedestal plasma transport