Plasma flow healing of magnetic islands in stellarators*

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Recent experiments from the Large Helical Device (LHD) indicate that the plasma flow can play a primary role in eliminating magnetic islands produced in the vacuum configuration of conventional stellarators. The observed island healing occurs at a critical plasma β that varies with plasma collisionality. A model explaining this phenomenon is developed based upon self-consistent island evolution and torque balance equations. In conventional stellarators, neoclassical damping physics plays an essential role in establishing the flow profiles. The balance of neoclassical damping and cross-field viscosity produces a radial boundary layer for the plasma rotation profile outside the separatrix of a locked magnetic island which decreases as the collisionality drops. This has the consequence of enhancing healing viscous torques at low collisionality making healing magnetic islands occur more readily in high temperature conventional stellarators.

I. Introduction

Magnetic island physics has been a major topic of interest to the stellarator community. There is no general solution to the 3-D magnetostatic equilibrium equations that guarantee the existence of topologically toroidal magnetic surfaces. Pressure induced magnetic islands and their subsequent overlap is thought to be a primary mechanism for equilibrium β limits. As such, 3-D equilibrium codes are often implemented in an effort to eliminate island formation in optimizing stellarator configurations [1–3]. However, recent studies on the Large Helical Device (LHD) have indicated that the elimination of imposed vacuum magnetic islands is observed [4] and correlated with this island healing are abrupt changes to the plasma flow [5]. These observations can be explained by a recent theoretical advance that models self-consistent island evolution and torque balance relations by accounting for neoclassical flow damping physics, cross-field viscosity and electromagnetic effects [6]

In the LHD experiments, external 3-D coils are intentionally applied to produce a magnetic island chain at a low-order rational surface of the vacuum configuration. Finite β plasmas are produced by neutral beam injection. A dependence of the magnetic island evolution on plasma parameters is observed by varying the field strength, density and heating power. As shown in Fig. 1, both magnetic island growth and healing is seen with the two disparate plasma responses distinguished by a sharp boundary in a parameter space defined by the plasma β and collisionality at the rational surface. For a given collisionality, there exists a critical β for island healing. This critical β drops as the collisionality at the rational surface decreases. Associated with the loss of the magnetic island is an abrupt change in the rotation profile [5]. In the presence of a locked island, the plasma rotation is inhibited at the rational surface. After the transition to the no island state, rotation is observed at the rational surface.

These results can be understood by considering the coupled torque balance and island evolution equations [6]. In the following, a theoretical formulation for use in interpreting the experimental results is developed. The theory generalizes previous analytic calculations in cylindrical geometry [7]. In the following calculation, particular emphasis is made on the important role neoclassical physics has in describing the flow properties of conventional stellarators.

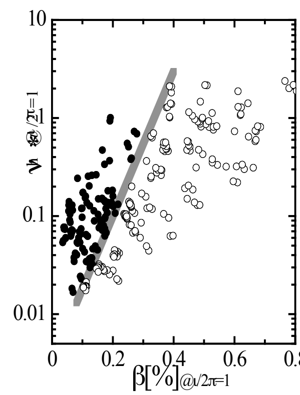

FIG. 1: From Narushima et al, 2008 *Nucl. Fusion* **48** 075010. Island growth and island healing are distinguished in a parameter space defined by the collisionality (ν^*) and plasma β at the rational surface. Dark circles denote island growth and open circles denote island healing.

The observations from the LHD island experiments have similiarities to results from tokamak [8] and reversed field pinch experiments [9] in the presence of externally produced resonant magnetic fields. Intrinsic or applied 3-D resonant external magnetic perturbations affects the growth and rotation properties of magnetic islands associated with the resonant surface. In tearing stable tokamaks, the problem that has governed the most attention is the field error penetration problem whereby an external resonant magnetic field can provide a source for producing a magnetic island. The penetration of this field is inhibited by plasma rotation. However, at sufficiently large 3-D field amplitude, the resonant field produces a forced reconnection at the rational surface. Associated with mode penetration is a change in the rotation profile with rotation at the rational surface abruptly changing to zero. Mode penetration is to be avoided in tokamaks as it often leads to disruptions.

The problem of interest here is an explanation for the reverse process, the elimination or healing of magnetic islands by the plasma flow. For this effort, a theoretical paradigm for understanding the interaction of tearing modes with resonant 3-D magnetic fields produced from an external source in cylindrical plasmas [7] can be adapted for stellarator applications. In this work, the amplitude and phase of the magnetic island is determined by coupled electromagnetic and fluid flow information. The conventional matched asymptotics procedure is used to calculate island evolution properties. The plasma rotation properties enter through a torque balance equation at the rational surface.

II. Magnetic Fields

The magnetic field is written as the sum of an equilibrium field with robust magnetic surfaces and an island producing magnetic field.

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1. \tag{1}$$

Here $\mathbf{B}_0$ satisfies $\mathbf{B}_0 \cdot \nabla\psi = 0$ where ψ labels topologically toroidal magnetic surfaces. The formation of an island at the rational surface $\iota_o = n_o/m_o$ is considered due to a magnetic field of the form $\mathbf{B}_1 \cdot \nabla\psi/\mathbf{B}_0 \cdot \nabla\phi = m_o A_o \sin(n_o\phi - m_o\theta)$ where $\theta(\zeta)$ denote the poloidal (toroidal) straight-field line angle. It is convenient to switch coordinates from (ψ, θ, ϕ) to (ρ, α, ζ) given by

$$\alpha = \theta - \iota_o\phi, \qquad \zeta = \phi, \tag{2}$$

and ρ a radial-like variable derived from $\psi = \psi(\rho) \sim \rho^2$. With $\mathbf{B}_1 = \nabla\chi\times\nabla\alpha + \nabla A\times\nabla\zeta$ [6], the total field can be written using the (ρ, α, ζ) coordinate system in the form

$$\mathbf{B} = \nabla[\psi(\rho) + \chi] \times \nabla\alpha + \psi'\nabla\zeta \times \nabla\Psi^*, \tag{3}$$

where $\psi' \equiv d\psi/d\rho(\rho = \rho_o)$ and the helical flux function Ψ^* described magnetic surfaces in the island region $\mathbf{B} \cdot \nabla\overline{\Psi^*} = 0$. Using a single harmonic approximation for $\overline{A} = \overline{A}(\rho_o, \alpha) = A_o \cos(m_o\alpha)$, the helical flux function describing the magnetic island flux surfaces is written

$$\overline{\Psi^*} = \frac{1}{2}\iota_o' x^2 - \frac{A_o}{\psi'}\cos(m_o\alpha), \tag{4}$$

where $\iota_o' \equiv d\iota/d\rho(\rho = \rho_o)$, $x = \rho - \rho_o$. and $\overline{A} = \oint d\zeta/2\pi A(\rho, \alpha, \zeta)$. From Eq. (4), the island width can be derived and is given by

$$w = 4\sqrt{\left|\frac{A_o}{\iota_o'\psi'}\right|}. \tag{5}$$

In writing Eqs. (3) and (4), a small island width expansion ($\delta = w/L_{eq} \ll 1$) is utilized.

In the absence of any plasma response, the perturbed fields $\mathbf{B}_1$ are solutions to the vacuum equations ($\nabla\cdot\mathbf{B}_1 = \nabla \times \mathbf{B}_1 = 0$) subject to a boundary condition describing the external source. The vacuum island width can be defined again assuming a dominant single harmonic approximation to the vacuum solution, $\overline{A^V} = A_o^V \cos(m_o\alpha + \Delta\phi)$ with the vacuum island width given by

$$w^V = 4\sqrt{\left|\frac{A_o^V}{\iota_o'\psi'}\right|}. \tag{6}$$

The quantity $\Delta\phi$ denotes the phase of the vacuum island relative to phase of the magnetic island when plasma is present.

The plasma response of interest is due to localized currents in the vicinity of the rational surface. Analytic island theories for stellarator applications strongly resemble that used in nonlinear tearing mode problems. As in tearing mode theory, the plasma response is treated differently in two distinct plasma regions and matched asymptotically. The strength of the localized currents as described by exterior region are quantified by the asymptotic matching parameter

$$\Delta'_{mn} = \frac{1}{A_{mn}(\rho_o)}\frac{dA_{mn}}{d\rho}\Big|_{\rho_o-}^{\rho_o+}. \tag{7}$$

In general three-dimensional systems, the matching data is significantly complicated by the degree of coupling between different rational surfaces. In order to make analytic progress in the following, the responses of each of the coupled surfaces is assumed negligible. As such, the dynamics of a single isolated island chain at the $\iota_o = n_o/m_o$ is emphasized. The exterior region solution is governed by a linear equation with the effect of the exterior source entering through a boundary condition. In particular, one can write $\Delta'_{m_o n_o} = \Delta'_o + \Delta'_{BC}$ where Δ'_o represents the asymptotic matching data in the absence of the external source and describes the inherent stability properties of the plasma and Δ'_{BC} is proportional to A^V. Additionally, a finite phase shift between the magnetic field associated with the locked island at the rational surface and the external source is allowed. The 'cosine' component corresponds to a contribution that affects the nonlinear island width and the 'sine' component enters into torque balance. Denoting these two different components Δ'_c and Δ'_s, we have

$$\Delta'_c = \Delta'_o[1 - k_v\frac{A^V}{A_o}\cos(\Delta\phi)], \tag{8}$$

$$\Delta'_s = k_v\Delta'_o\frac{A^V}{A_o}\sin(\Delta\phi), \tag{9}$$

where k_v is an order unity parameter that is a function of the plasma equilibrium. In vacuum, $k_v = 1$. As viewed from the exterior region, the quantities Δ'_c and Δ'_s denote localized current responses at the rational surface that denote the asymptotic behavior of the currents in the exterior region. The conventional matching conditions of tearing mode theory are now identified.

$$-\int dx \int_0^{2\pi} \frac{d\alpha}{\pi} \int_0^{2\pi} \frac{d\zeta}{2\pi} \cos(m_o\alpha) \frac{\mu_o\gamma}{g^{\rho\rho}} \frac{\mathbf{J}_1 \cdot \mathbf{B}_0}{B^2} = \Delta'_c A_o, \tag{10}$$

$$-\int dx \int_0^{2\pi} \frac{d\alpha}{\pi} \int_0^{2\pi} \frac{d\zeta}{2\pi} \sin(m_o\alpha) \frac{\mu_o\gamma}{g^{\rho\rho}} \frac{\mathbf{J}_1 \cdot \mathbf{B}_0}{B^2} = \Delta'_s A_o, \tag{11}$$

where the island resolved currents used to describe the plasma response are inserted in the integrals. Note that in the vaccum limit ($\mathbf{J}_1 \cdot \mathbf{B}_0 = 0$), $\Delta'_c = \Delta'_s = 0$ describes the expected solution $\Delta\phi = 0$ and $A = A^V$.

For the standard tearing mode approach, a boundary layer theory is used to describe solutions in the vicinity of the island where ρ derivatives on perturbed quantities are assumed large. Currents are calculated in the island region are then used in the matching conditions, Eqs. (10) and (11). The primary problem of interest is the suppression of vacuum islands which are typically large compared to resistive layer scales [4, 5]. As such, Rutherford theory is the appropriate approach to follow for the 'cosine' component of current which describes nonlinear evolution of the island width [10]. For simplicity a number of finite β effects [11, 12] are neglected in the derivation of the generalized Rutherford equation. From the a resistive Ohm's law, ($\mathbf{E} \cdot \mathbf{B} = \eta \mathbf{J} \cdot \mathbf{B}$) the standard Rutherford results is derived $k_1(\mu_o/\eta)dw/dt = \Delta'_c$ where η is the plasma resistivity and $k_1 \approx 0.8$. Island saturation ($dw/dt = 0$)is given by $\Delta'_c = 0$ which produces the island width expression

$$w = w^V \sqrt{k_v \cos(\Delta\phi)}. \tag{12}$$

The complete theory requires an expression for $\Delta\phi$

The 'sine' component of the parallel current is associated with the presence of a net electromagnetic torque on the plasma. A connection can be made between the asymptotic matching condition Eq. (11) and torque balance [6]. To show this, we first define the flux surface averaged torques as

$$T_{EM\alpha} = \int_0^{2\pi} \int_0^{2\pi} \mathbf{e}_\alpha \cdot \mathbf{J} \times \mathbf{B} \frac{\sqrt{g}}{g^{\rho\rho}} d\zeta d\alpha, \tag{13}$$

In the exterior region where ideal MHD governs the dynamics, these torques are identically zero. Hence, $T_{EM\alpha}$ is localized to the island region. From the magnetic field representation and radial force, one can derive [6]

$$T_{EM\alpha} = \frac{2\pi\psi'}{\mu_o\gamma} m_o A_o \int_0^{2\pi} d\alpha \sin(m_o\alpha) \frac{\partial^2 \overline{A}}{\partial\rho^2}. \tag{14}$$

Integrating this equation in ρ to obtain T_{EM0} one finds

$$T_{EM0} \equiv \int dx\, T_{EM\alpha} = \frac{2\pi^2\psi'}{\mu_o\gamma} m_o A_o^2 \Delta'_s, \tag{15}$$

which demonstrates a relationship betweem the asymptotic matching condition defined in Eq. (11) and the net electromagnetic torque on the plasma. Using Eqs. (6), (9) and (12), an expression for the electromagnetic torque can be derived

$$T_{EM0} = \frac{\pi^2\psi'^3}{\mu_o\gamma} \frac{m_o k_v^2(-\Delta'_o)}{256} (\iota'_o)^2 (w^V)^4 \sin(2\Delta\phi). \tag{16}$$

This expression describes the integrated torque over the island region produced by the external resonant magnetic field as a function of the phase shift $\Delta\phi$. A self-consistent for $\Delta\phi$ is obtained by calculating Δ_s from the 'sine' component of the parallel current in the island region.

III. Plasma Flows and Viscous Torques

The 'sine' component of parallel current in the island region is typically associated with dissipative physics in steady state. For sufficiently large islands, density, temperature and electrostatic potential profiles are constant within the island region but nonzero outside the separatrix. These gradients are associated with flow velocities that influence the island rotation properties.

For nonlinear islands, no net plasma flow is allowed through the separatrix. This is often referred to as the no-slip constraint [7]. Since the dominant dissipation in the model to follow is that described by the ion-root dominant neoclassical damping and cross-field viscous forces, the phase velocity of the island is determined by the ion rotation rate. However, electron diamagnetic effects can affect the island phase velocity in more sophisticated plasma models [12].

The no-slip constraint and the requirement that locked islands have zero phase velocity uniquely determine the plasma flow at the rational surface. The requirement that the flow be fixed at $\iota = n_o/m_o$ is generally not consistent with the value of the flow profile calculated from a transport equation that accounts for neoclassical and turbulent transport and momentum sources. Noting that the electromagnetic torque calculated in Eq. (16) is localized to the island region, steady state momentum balance is established by balancing this torque with a similarly localized viscous torque in the island region. The viscous torque is characterized by a phenomenological cross-field viscosity coefficient [7]. Torque balance determines a self-consistent value for $\Delta\phi$.

What is demonstrated in the following is that currents of relevance to the torque balance equation are due to properties of the plasma flows outside the island separatrix. The flow profile is established by a transport equation that accounts for the competition between neoclassical and viscous transport properties. Unlike tokamaks, neoclassical flow damping physics plays a primary role in determining the self-consistent flow profiles in conventional stellarators. Poloidal and toroidal rotation damping rates are typically comparable in conventional stellarators whereas there is generally a vast disparity of these two rates in tokamaks with weak 3-D magnetic fields [13]. This difference in the neoclassical physics translates to different amplitudes of the viscous torque in conventional stellarators relative to axisymmetric configurations. In particular, a radial boundary layer in the rotational profile in the vicinity of the magnetic island develops due to the competition between viscous and neoclassical flow effects. As the boundary layer shrinks, the viscous torque on the plasma becomes larger. The width of the boundary layer is determined by the strength of the non-ambipolar neoclassical transport. In high temperature stellarators, the flow damping rates scale inversely with the collision frequency (corresponding the $1/\nu$ regime of neoclassical transport theory [14]). Hence, a smaller collisionality produces a larger viscous force. For the same natural rotational value and external field strength, it is easier to heal magnetic islands in a conventional stellarator than in the corresponding tokamak.

To describe the evolution of the flow profile, a momentum balance equation of the form

$$\rho_M \frac{d\mathbf{v}}{dt} = \mathbf{J}\times\mathbf{B} - \nabla p - \nabla\cdot\vec{\vec{\pi}} + \nabla\cdot(\rho_M \nu_\perp \nabla\mathbf{v}) + \mathbf{S}, \quad (17)$$

is used where ρ_M is the mass density, $\vec{\vec{\pi}}$ the viscous stress tensor and $\mathbf{S}$ external momentum sources. The cross-field viscous force uses a phenomenological viscosity coefficient $\nu_\perp$ meant to model turbulent and collisional processes that are not described by $\nabla\cdot\vec{\vec{\pi}}$.

Before preceding to the island case, let's consider the no island limit first. For this case, $\mathbf{B}\cdot\nabla\rho = 0$ and the lowest order momentum balance equation $\mathbf{E}+\mathbf{v}\times\mathbf{B} = \nabla p_i/n_i q_i$ allows one to write the flows

$$\mathbf{v} = (\frac{d\Phi}{d\rho} + \frac{1}{n_i q_i}\frac{dp_i}{d\rho})\frac{\mathbf{B}\times\nabla\rho}{B^2} + \frac{v_{||}}{B}\mathbf{B}, \quad (18)$$

where Φ is the electrostatic potential and p_i, n_i and q_i are the ion's pressure, density and charge. Alternatively, the flows can be written using the coordinate system given by Eq. (2)

$$\mathbf{v} = \Omega^\alpha \mathbf{e}_\alpha + \Omega^\zeta \mathbf{e}_\zeta, \quad (19)$$

where Ω^α and Ω^ζ are related to $\Phi' = d\Phi/d\rho$, $p_i' = dp_i/d\rho$ and $v_{||}$ by

$$\Omega^\alpha = \frac{1}{\psi'}(\Phi' + \frac{p_i'}{n_i q_i}) + \mathcal{O}(\delta), \quad (20)$$

$$\Omega^\zeta = -\frac{I}{\gamma}\Omega^\alpha + \frac{v_{||}B}{\gamma}. \quad (21)$$

A transport equations for the steady state flow profiles are established by taking the flux surface of $\mathbf{e}_i$ components of the next order momentum balance equation. In particular, the flux surface average of the $\mathbf{e}_\alpha$ projection of the steady state momentum balance yields

$$0 = -\psi' < \mathbf{J}\cdot\nabla\rho >_o - < \mathbf{e}_\alpha\cdot\nabla\cdot\vec{\vec{P}} >_o + < \mathbf{e}_\alpha\cdot\nabla\cdot(\rho_M\nu_\perp\nabla\mathbf{v}) >_o + < \mathbf{e}_\alpha\cdot\mathbf{S} >_o, \quad (22)$$

where $\vec{\vec{P}} = p\vec{\vec{I}} + \vec{\vec{\pi}}$ and

$$< f >_o = \frac{1}{V'}\int_0^{2\pi}\int_0^{2\pi} d\alpha d\zeta \sqrt{g} f, \quad (23)$$

and $V' = \int_0^{2\pi}\int_0^{2\pi} d\alpha d\zeta\sqrt{g}$. Here, the general function f is expressed as a function of ρ, α and ζ so that $< f >_o$ is purely a function of ρ. By requiring $< \mathbf{J}\cdot\nabla\rho >_o = 0$ in steady state, the above expression can be used to determine the radial electric field profile from the combination of cross-field viscosity, neoclassical damping and sources. The second term in Eq. (22) describes neoclassical transport in three-dimensional configurations and can be identified with the non-ambipolar component of the particle flux.

$$< \mathbf{e}_\alpha\cdot\nabla\cdot\vec{\vec{P}} >_o = -\psi'\sum_s q_s < \vec{\Gamma}_s^{neo}\cdot\nabla\rho >_o . \quad (24)$$

In conventional stellarators, generally the requirement of non-ambipolar neoclassical transport $\vec{\Gamma}_i^{neo} = \vec{\Gamma}_e^{neo}$ produces an equation for the radial electric field or alternatively Ω^α as functions of the plasma density and temperature gradients. In stellarator configurations, due to the lack of intrinsic ambipolar transport, mulitple solutions to the radial electric field profile can be obtained [15, 16] which complicate this analysis. Nevertheless, formally a solution to the equilibrium flow profile transport equations can be deduced.

To make analytic progress, we specialize our calculation to regimes of interest in high temperature conventional stellarators. In the absense of a radial electric field, the neoclassical transport is in the $1/\nu$ regime where the cross-field transport coeffients scale inversely with collision frequency. As such, the neoclassical transport can be written

$$\begin{aligned} & e < \vec{\Gamma}_s^{neo}\cdot\nabla\rho >_o \\ &= -C_{1/\nu}\rho_M\frac{\omega_{ts}^2}{\nu_s} < \frac{g^{\rho\rho}}{B^2} >_o [\Phi'\frac{q_s}{e} + \frac{p_s'}{n_s e} + k_s\frac{T_s'}{e}] \\ &= -C_{1/\nu}\rho_M\frac{\omega_{ts}^2}{\nu_s} < \frac{\psi' g^{\rho\rho}}{B^2} >_o \frac{q_s}{e}[\Omega^\alpha - \Omega^\alpha_{amb,s}], \quad (25)\end{aligned}$$

where ν_s is the collision frequency, $\omega_{ts} = v_{ts}/R_o$ is the transit frequency with v_{ts} the thermal velocity of species s, R_o the major radius, $T_s' = dT_s/d\rho$ and $k_s \sim 1$. The dimensionless parameter $C_{1/\nu}$ in the zero radial electric field limit scales as $C_{1/\nu} \sim \epsilon_{eff}^{3/2}$ where ϵ_{eff} is a measure of the effective helical ripple [17]. These high transport rates are generally reduced in the presence of a sufficiently large radial electric field (or Ω^α). However, since the problem of interest is a description of the flow profile in the vicinity of a locked island, these electric field corrections are ignored for simplicity.

Using the expression as given in the second form for Γ_s^{neo} demonstrates that Eq. (22) can by viewed as a transport equation for Ω^α as a function of the sources and equilibrium density and temperature profiles.

The density, temperature and electrostatic potential profiles equilibrate on the helical magnetic surfaces of the island $\overline{\Psi}^*$ given in Eq. (4) for sufficiently large magnetic island width. This result comes from the leading order parallel Ohm's law, density and pressure evolution which yield $\mathbf{B}\cdot\nabla\Phi+\mathbf{B}\cdot\nabla p_e/n_e e=0, \mathbf{v}_E\cdot\nabla n=\mathbf{v}_E\cdot\nabla p_s=0$. In this limit, Ω^α is given by

$$\Omega^\alpha = \frac{1}{\psi'}[\frac{d\Phi}{d\overline{\Psi}^*}+\frac{1}{n_i q_i}\frac{dp_i}{d\overline{\Psi}*}]\iota' x =<\Omega^\alpha>\frac{x}{<x>}, \quad (26)$$

where the flux surface averaging procedure in the presence of an island is given by

$$<f>=\frac{\int_0^{2\pi}\int_0^{2\pi}d\alpha d\zeta\sqrt{g}x^{-1}f(\overline{\Psi}^*,\alpha,\zeta)}{\int_0^{2\pi}\int_0^{2\pi}d\alpha d\zeta\sqrt{g}x^{-1}}, \quad (27)$$

with $<f>$ being a function of $\overline{\Psi}^*$ only. In this expression all quantities are expressed as functions of $\overline{\Psi}^*, \alpha$ and ζ.

To calculate a transport equation, the same procedure used to construct Eq. (22) can be followed with the result

$$0=-\frac{\psi'}{\iota'}<\mathbf{J}\cdot\nabla\overline{\Psi}^*>-<x\mathbf{e}_\alpha\cdot\nabla\cdot\vec{\vec{P}}> + <x\mathbf{e}_\alpha\cdot\nabla\cdot(\rho_M\nu_\perp\nabla\mathbf{v})>+<x\mathbf{e}_\alpha\cdot\mathbf{S}>. \quad (28)$$

By setting $<\mathbf{J}\cdot\nabla\overline{\Psi}^*>=0$, a transport equation for $<\Omega^\alpha>$ is obtained. In the vicinity of the magnetic island, this equation can be written

$$<\Omega^\alpha>=\Omega_0^\alpha +\delta_r^2\frac{\iota'^2<x>^2}{c_r<x^2g^{\rho\rho}/B^2>}\frac{d}{d\Psi^*}[\frac{<x^4(g^{\rho\rho})^2/B^2>}{<x>}\frac{d}{d\Psi^*}\frac{<\Omega^\alpha>}{<x>}], \quad (29)$$

where $c_r=<(g^{\rho\rho})^2/B^2>/<g^{\rho\rho}/B^2>$ and Ω_0^α given by

$$\Omega_0^\alpha=<\Omega_{amb,i}^\alpha>+\frac{\nu_i<x\mathbf{e}_\alpha\cdot\mathbf{S}><x>}{\omega_{ti}^2\rho_M C_{1/\nu_i}<x^2\psi'^2g^{\rho\rho}/B^2>}, \quad (30)$$

is the rotation profile as determined by sources and neoclassical transport. An important parameter in Eq. (29) is the distance δ_r defined by

$$\delta_r^2=\frac{c_r\nu_\perp\nu_i}{\omega_{ti}^2C_{1/\nu_i}}, \quad (31)$$

which is a measure of the cross-field viscosity relative to the strength of the neoclassical damping. In writing down Eq. (29), the 'ion root' is assumed which is the relevant case for solutions with small E_r. In writing the above, it is assumed that the basic functional form for the neoclassical transport is unaffected by the presence of the island. However, the coefficient for the neoclassical transport $C_{1/\nu}$ is modified by the island [18].

For large enough ϵ_{eff}, the inequality $\delta \ll L_{eq}$ is expected noting the prominence of neoclassical physics. In this case, the cross-viscosity plays a small role and the solution $<\Omega^\alpha>=\Omega_0^\alpha$ is predicted. However, with a locked island, the rotation at the island separatrix $<\Omega^\alpha(\overline{\Psi}^*=\overline{\Psi}^*_{sx})>$ must be consistent with the no-slip constraint. This condition provides an internal boundary condition for the flow profile which is generally in conflict with the zero δ_r solution described above. As such, a boundary layer of width δ_r develops outside of the separatrix. In the simple asymptotic limit limit $\delta_r\sim X_*\gg w$, the transport equation Eq. (29) reduces to

$$<\Omega^\alpha>=\Omega_0^\alpha+\delta_r^2\frac{d^2<\Omega^\alpha>}{dX_*^2}, \quad (32)$$

where the radial-like coordinate X_* is defined by $\overline{\Psi}^*-\overline{\Psi}^*_{sx}=\iota' X_*^2/2$. The solution to this expression subject to the locked island boundary condition described above is given by

$$<\Omega^\alpha>=\Omega_0^\alpha(1-\ e^{-|X_*|/\delta_r}), \quad (33)$$

For the more general case where the island width is not assumed small, there is no simple analytic solution for Eq. (29). Nonetheless, it is clear that δ_r sets the radial scale for the boundary layer solution for the flow profile. Hence, we can formally write the solution to Eq. (29) as

$$<\Omega^\alpha>=\Omega_0^\alpha[1-f_r(\frac{X_*}{\delta_r},\frac{w}{\delta_r})], \quad (34)$$

with the asymptotic solution $f_r=e^{-|X_*|/\delta_r}$ in the small island limit. The flow profile described in Eq. (34) is similar to the equivalent locked island calculation for the toroidal flow velocity in tokamaks when neoclassical toroidal viscosity is present [19].

The calculated flow profiles produce perpendicular currents from the momentum balance equation. When coupled with the quasineutrality condition $\mathbf{B}\cdot\nabla(\mathbf{J}_1\cdot\mathbf{B}_0/B^2)=-\nabla\cdot\mathbf{J}_\perp$, the flow effects produce parallel currents that enter into the torque balance relation. The condition of interest is Eqs. (11) which can be written in the form

$$T_{EM0}=\frac{2\psi'}{\iota'}\int_{-A_o}^{\infty}d\overline{\Psi^*}\int d\alpha\int d\zeta\frac{\sqrt{g}}{g^{\rho\rho}}\nabla\cdot\mathbf{J}_\perp. \quad (35)$$

Using the momentum balance equation to determine $\mathbf{J}_\perp$, and symmetry arguments, the torque balance equation is given by

$$T_{EM0}=\int_{-A_o}^{\infty}d\overline{\Psi^*}\frac{2V'}{\iota'<x>}(<\frac{\mathbf{e}_\alpha\cdot\nabla\cdot\vec{\vec{\pi}}}{g^{\rho\rho}}>-<\frac{\mathbf{e}_\alpha\cdot\mathbf{S}}{g^{\rho\rho}}>). \quad (36)$$

Using properties of the model previously discussed and the solution to the transport equation for Ω^α, Eq. (34), one obtains

$$T_{EM0}=\int_{A_o}^{\infty}d\overline{\Psi}^*[-\frac{2V'C_{1/\nu}\rho_M\omega_{ti}^2<\psi'^2x/B^2>}{\nu_i\iota'<x>^2}\Omega_0^\alpha f_r -\frac{2V'}{\iota'<x>}<\frac{\mathbf{e}_\alpha\cdot\mathbf{S}}{g^{\rho\rho}}(1-\frac{xg^{\rho\rho}<x/B^2>}{<x^2g^{\rho\rho}>})>]. \quad (37)$$

If the source function is taken to be small in the island region and the resonant component of the magnetic spectrum is small (so that $< x/B^2 > / < x >\approx< 1/B^2 >$), the important spatial dependence of the integrand is contained in the function f_r describing the boundary layer solution for the flow profile in Eq. (34). This function effectively limits the radial extent of the integration to the characteristic scale δ_r. Utilizing this, the torque balance condition $T_{EM0} + T_{V0} = 0$ can be derived with the viscous torque described by

$$T_{V0} = 2k_f V' \rho_M \nu_\perp c_r < \frac{\psi'^2}{B^2} > \frac{\Omega_0^\alpha}{\delta_r}, \quad (38)$$

where the dimensionless coefficient k_f is defined as a function of the quantity $z = w/\delta_r$ by

$$k_f(z) = \int_0^\infty dy f_r(y,z) \frac{2K(k)}{\pi} \frac{y}{\sqrt{y^2 + z^2/4}}, \quad (39)$$

where $K(k)$ is the complete elliptic integral of the first kind, $k = \sqrt{z^2/(4y^2+z^2)}$ and f_r is defined with respect to the variable $y = X_*/\delta_r$. In the limit $z = 0, k_f = 1$.

While T_{V0} is used to denote the 'viscous' torque, it should be noted that both cross-field viscosity and neoclassical damping physics are used to calculate this contribution to the steady-state torque balance relation [20].

IV. Healing a locked island

With the information derived above, we are now in a position to predict the conditions for healing a locked island. The torque balance equation $T_{EM0} + T_{V0} = 0$ and and Eqs. (16) and (38), one can write the equation

$$D_A \sin(2\Delta\phi) = D_\Omega, \quad (40)$$

where

$$D_A = \omega_A^2 \frac{m_o^2 k_v^2}{k_f c_r} \frac{(-\rho_o \Delta'_o)}{2m_o} \frac{(\iota'_o)^2 (w^V)^4}{512}, \quad (41)$$

$$D_\Omega = -\nu_\perp \rho_o \frac{d\Omega^\alpha}{d\rho}\Big|_{\rho_o-}^{\rho_o+} = -2\nu_\perp \frac{\rho_o}{\delta_r} \Omega_0^\alpha, \quad (42)$$

and $\omega_A^2 = 4\pi^2 \psi'/V' \gamma \rho_M \mu_o < 1/B^2 >\approx v_A^2/R_o^2$ is the Alfven frequency.

In the absence of a plasma response $D_\Omega = 0$, the island is locked to the external source with $\Delta\phi = 0$. As D_Ω rises, a finite phase shift is predicted given by $\sin(2\Delta\phi) = D_\Omega/D_w$. With yet larger $|D_\Omega|$, the amplitude of the phase shift becomes larger until $|\sin(2\Delta\phi)|$ reaches its maximal value at unity corresponding to $|\Delta\phi| = \pi/4$. At this point $|D_\Omega| = D_w$. At higher $|D_\Omega| > D_A$, steady state momentum balance can no longer be satisfied. At this point, finite inertia now plays a role and the torque balance equation becomes

$$\int dx \int_0^{2\pi} \int_0^{2\pi} \sqrt{g} d\alpha d\zeta\ \rho_M \frac{\partial \Omega^\alpha}{\partial t} \frac{\psi'^2}{B^2} = T_{EM0} + T_{V0}. \quad (43)$$

When $|D_\Omega| > D_w$, the viscous torque overwhelms the electromagnetic torque and the plasma at $\rho = \rho_o$ starts to rotate. At this point, the island is no longer locked to the wall. Eddy currents are produced at the rational surface that shield the external magnetic field from penetrating; the large magnetic island disappears. The condition $|D_\Omega| = D_w$ is the criteria for healing a magnetic island with plasma flow.

The flow healing criteria can be used to predict a critical β for island suppression as a function of collisionality. The collisionality dependence enters naturally through the flow boundary layer solution described in Eq. (34). For scaling arguments, we require a model for the cross-field viscosity. Using a gyro-Bohm scaling for cross-field viscosity [$\nu_\perp \sim\sim (\rho_i/L_{eq})(T/eB)$ where ρ_i is the ion gyro-radius] and diamagnetic level flows, the healing criteria $|D_\Omega| = D_w$ produces a scaling for the critical β given by

$$\beta_{crit} \sim (\nu^*)^{1/4} (\frac{w^V}{L_{eq}})^2 \frac{L_{eq}\omega_{pi}}{C_{1/\nu}^{1/4} c}, \quad (44)$$

where $\nu^* = \epsilon^{3/2}\nu_i/\omega_{ti}$ is the normalized collisionality and $\omega_{pi}/c = \sqrt{ne^2\mu_o/m_i}$ is the ion skin depth. For fixed vacuum island width, the above scaling indicates that the critical β for island healing scales weakly but monotonically with collisionality. This result is qualitatively consistent with the results from LHD [4, 5] shown in Fig. 1.

V. Summary and discussion

In this work, a theory describing the interaction of a tearing stable conventional stellarator with an externally produced resonant magnetic field perturbation is provided. The calculation highlights the important role plasma rotation can have in healing vacuum magnetic islands. The model used is a stellarator-specific extension of an earlier theoretical effort describing the interaction of tearing modes with resonant magnetic perturbations in cylindrical geometry [7]. In this work, transitions between different asymptotic states of the plasma are described by coupled electromagnetic and fluid flow physics.

The results of the present model can used to interpret the island healing experiments of LHD [4, 5]. In particular, this model explains the basic phenomenology of how plasma flow physics can heal islands in stellarators. The model provides a theoretical explanation for the empirically derived scaling of the critical β as a function of plasma collisionality. Additionally, a hysteresis effect is predicted whereby the criteria for the reappearance of a healed island differs from the criteria for island healing [6].

An important distinction between conventional stellarators and tokamaks lies in the treatment of neoclassical physics. In high temperature conventional stellarators, the viscous force associated with non-ambipolar neoclassical transport plays an important role in establishing the flow profile in the vicinity of a locked mode. A model for the rotation profile is provided that accounts for a balance between cross-field viscosity and neoclassical flow damping. The presence of $1/\nu$ neoclassical transport in high temperature conventional stellarators produces a radial boundary layer in the flow profile outside the separatrix of a locked island. This has the consequence of effectively raising the viscous torque as the plasma becomes less collisional. Therefore, island suppression by plasma flow becomes easier at lower collisionality.

The healing criteria is given by $|D_\Omega| = D_A$ where D_Ω and D_A are measures of the viscous and electromagnetic torques, respectively as described in Eqs. (41) and (42). This theory produces a critical β for island healing as a function of collisionality and external 3-D field resonant field amplitude described in Eq. (44). The scaling indicates a weak but monotonic dependence of the critical β for healing with plasma collisionality, a prediction which is in qualitative agreement with the experimental results from LHD [4, 5].

The neoclassical physics of conventional stellarators have been emphasized. For these devices, the flow damping rates in the toroidal and poloidal directions can formally have comparable values. The flow properties of quasisymmetric configurations are different than those considered here [21, 22]. These devices have weak neoclassical flow damping in the symmetry direction. The equivalent calculation for the healing of magnetic islands by plasma flows requires a procedure more akin to the theories used for axisymmetric (or nearly axisymmetric) tokamak plasmas [19]. One of the purported advantages of quasisymmetric stellarators is the presence of undamped flows in the symmetric directions which are known to have a variety of beneficial effects on stability and transport in tokamaks. However, for the conventional stellarator case discussed here, it is the presence of the $1/\nu$ transport and the associated neoclassical viscous force that enhances the viscous torque that heals the magnetic island.

An important implication of this work is that plasma rotation physics can play a crucial role on magnetic island physics and surface fragility in stellarator configurations. The requirement of ambipolar neoclassical transport or external momentum sources can produce self-consistent plasma flows of sufficient magnitude to eliminate vacuum magnetic islands. The physics of flow suppression is not incorporated in 3-D MHD equilibrium codes [1–3]. These tools are unduly pessimistic in predicting the topological breakup of magnetic surfaces. Initial value extended MHD codes are capable of describing magnetic island healing due to plasma flows. However, in order to obtain correct quantitative solution, extended MHD modeling would need to include 3-D field induced non-ambipolar neoclassical particle fluxes and the viscous forces they induce.

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