```
(* Mathematica notebook file ParNeoRes.pdf for calculating parallel neoclassical
 resistivity, which is also availsble as UW-CPTC 11-5 via http://www.cptc.wisc.edu.
   (Mathematica .nb file available upon request from callen@engr.wisc.edu.)

  J.D. Callen, July 11, 2011.

  This evaluation of the "local" parallel neoclassical resistivity on a magnetic flux
  surface is based mainly on inversion of the 2x2 matrices that result from the
  combination of electron parallel flow and heat flow equations developed in UW-
 CPTC 09-6R, which is available via http://www.cptc.wisc.edu,
or via "Article Objects," Supplementary Files (EPAPS) 033091php.pdf for Ref. [11]
 in J.D.Callen, C.C. Hegna, and A.J. Cole, Phys. Plasmas 17, 056113 (2010).

  Also, a comparison is made at the end with the approximate,
more phenomenological formula given in Eq. (9) in the "Model For Pedestal Structure"
 2011 Physics of Plasmas paper to which this supplementary file is attached,
which will be referred to below as UW-CPTC 11-4 (available via http://www.cptc.wisc.edu).

   Finally, the results of the 2x2 matrix inversion results
 are compared to the neoclassical parallel resistivity formulas in
 S.P. Hirshman, R.J. Hawryluk and B. Birge, Nucl. Fusion 17,
611 (1977) (often used in ONETWO) and
 O. Sauter, C. Angioni and Y.R. Lin-Liu, Phys. Plasmas 6, 2834 (1999).
*)


(* INPUT PLASMA PARAMETERS *)
(* The numbers used here are obtained at normalized rho_N = 0.98,
rho = 75.13 cm from the ONETWO outone and iterdb files for DIII-D discharge 98889
   and are as indicated in Nucl. Fusion 50, 064004 (2010),
except the Zeff value of 2.6 is estimated from J.M. Canik's SOLPS modeling
  [J.M. Canik et al., Phys. Plasmas 18, 056118 (2011)] using 4
 different carbon models for this pedestal, as reported in J.D. Callen,
J.M. Canik and S.P. Smith, "Pedestal Structure Model," report UW-CPTC 11-3,
July 2011 (submitted to Phys. Rev. Lett.)
*)

Te = 352.           (* electron temperature in eV *)

352.

ne = 1.77 × 10^(19)  (* electron density in m^{-3} *)

1.77 × 10^19

Zeff = 2.6  (* effective ion charge for electron scattering, Eq. (A8) in UW-CPTC 09-6R *)

2.6


(* INPUT DEVICE PARAMETERS *)

rM = 0.593           (* half-width of flux surface on mid-plane, in m *)

0.593

Rzero = 1.686        (* average major radius of flux surface in mid-plane, in m *)

1.686
```

```
epsilon = rM / Rzero (* magnetic inverse aspect ratio, Eq. (B6) in UW 09-6R *)
```

0.35172

```
q = 4.49            (* "safety factor" at this flux surface *)
```

4.49

```
(* SOME CONSTANTS OF NATURE *)
me = 9.1094 × 10^(-31)          (* electron mass in kg *)
```

$9.1094 \times 10^{-31}$

```
echarge = 1.6022 × 10^(-19 )    (* electron charge in Coulombs *)
```

$1.6022 \times 10^{-19}$

```
muzero = 4 × 3.14159 × 10^(-7)   (* permeability of vacuum *)
```

$1.25664 \times 10^{-6}$

```
(* FIRST, calculate ln Lambda -- see Eqs. (2.09)-(2.11) in Chapter 2,
available via http://homepages.cae.wisc.edu/~callen/book *)
DebyeL = 7434.0 Sqrt[Te / ne]      (* Debye length in m *)
```

0.0000331518

```
bmincl = 4.8 × 10^(-10) Zeff / Te   (* classical bmin in m *)
```

$3.54545 \times 10^{-12}$

```
bminqm = 1.1 × 10^(-10) / Sqrt[Te] (* quantum mechanical bmin in m *)
```

$5.86302 \times 10^{-12}$

```
lnLambda = Log[DebyeL / Max[bmincl, bminqm]] (* ln Lambda, dimensionless *)
```

15.5479

```
(* NEXT, calculate some key electron collisionality parameters *)
nue = 4.941 × 10^(-11) ne Zeff (lnLambda / 17.0) / Te^1.5
(* electron collison frequency nu_e in /s, Eq. (A9) in UW CPTC 09-6R *)
ScientificForm[%, 3]
```

314 899.

$3.15 \times 10^{5}$

```
vTe = Sqrt[ 2.0 echarge Te / me]
(* electron thermal velocity v_Te = sqrt{2 Te / me} in m/s *)
ScientificForm[%, 3]
```

$1.11276 \times 10^{7}$

$1.11 \times 10^{7}$

```
lambdae = vTe / nue
(* electron collision length lambda_e in m, Eq. (A10) in UW CPTC 09-6R *)
ScientificForm[%, 3]
```

35.3369

$3.53 \times 10^{1}$

```
omegate = vTe / (Rzero q)
(* electron transit frequency in /s, Eq. (B10)in UW CPTC 09-6R *)
ScientificForm[%, 3]
```

$1.46993 \times 10^{6}$

$1.47 \times 10^{6}$

```
nueOVERomegate = nue / omegate
(* ratio of electron collision to transit frequency *)
ScientificForm[%, 3]
```

0.214228

$2.14 \times 10^{-1}$

```
USUALnustare = nue / (epsilon^1.5 omegate)
(* Usual electron collisionality parameter nu_*e = nu_e / epsilon^{3/2} omegate *)
ScientificForm[%, 3]
```

1.02702

1.03

```
fc = (1.0 - epsilon) ^2 / (Sqrt[1.0 - epsilon^2] (1.0 + 1.46 Sqrt[epsilon]
       + 0.2 epsilon))
(* flow-weighted fraction of circulating particles,
most rigorously calculated via first form in Eq. (B5) in UW CPTC 09-6R
  or first form in Eq. (6) in UW-CPTC 11-4;
Pade approximate formula here is f_c^p from Eq. (B9) in C.T. Hsu,
K.C. Shaing, R.P. Gormley and D.J. Sigmar, Phys. Fluids B 4, 4023 (1992)
*)
```

0.231872

```
ft = 1.0 - fc    (* flow-weighted fraction of trapped particles *)
```

0.768128

```
nustare = (ft / (1.46 fc epsilon^2)) (nue / omegate)
(* neoclassical electron collisionality parameter,
most rigorously calculated via first form in Eq. (B12) in UW-CPTC 09-6R
  or Eq. (7) in UW-CPTC 11-4;
approximate form here uses geometric factor = epsilon^2/ (2 Rzero^2 q^2) *)
```

3.92929

```
(* ALSO, calculate dimensionless viscosity components from Table 1 in UW CPTC 09-6R *)
K00B = (Zeff + 0.533) / Zeff
```

1.205

```
K01B = (Zeff + 0.707) / Zeff
```

1.27192

```
K11B = (2.0 Zeff + 1.591) / Zeff

2.61192

K00P = 1.77

1.77

K01P = 5.32

5.32

K11P = 21.27

21.27

Denom = 2.4 Zeff^2 + 5.32 Zeff + 2.225

32.281

K00PS = (4.25 Zeff + 3.02) / Denom

0.43586

K01PS = (20.13 Zeff + 12.43) / Denom

2.00638

K11PS = (101.06 Zeff + 58.65) / Denom

9.95651


(* THEN, calculate dimensionless multi-
 collsionality viscosity cefficients as defined in Eq. (B13) in UW CPTC 09-6R *)

omegatau = Zeff omegate / nue
(* dimensionless omega_te tau_ee parameter in Eq. (B13) in UW CPTC 09-6R *)

12.1366

K00tot = K00B /
  ((1.0 + Sqrt[nustare] + 2.92 nustare K00B / K00P) (1.0 + 2.0 K00P / (3.0 omegatau K00PS)))

0.0912812

K01tot = K01B /
  ((1.0 + Sqrt[nustare] + 2.92 nustare K01B / K01P) (1.0 + 2.0 K01P / (3.0 omegatau K01PS)))

0.193912

K11tot = K11B /
  ((1.0 + Sqrt[nustare] + 2.92 nustare K11B / K11P) (1.0 + 2.0 K11P / (3.0 omegatau K11PS)))

0.532343


(* AND THEN calculate the dimensionless viscosity matrix M_e,
Eq. (C11) in UW CPTC 09-6R *)
Me = Zeff (ft / fc)
  {{K00tot, 2.5 K00tot - K01tot}, {2.5 K00tot - K01tot, K11tot - 5.0 K01tot + 6.25 K00tot}}

{{0.786214, 0.29535}, {0.29535, 1.14803}}
```

```
(* AND ALSO calculate the dimensionless friction matrix N_e, Eq. (C9) in UW CPTC 09-6R *)
Ne = {{Zeff, 1.5 Zeff}, {1.5 Zeff, 1.414 + 3.25 Zeff}}
```

{{2.6, 3.9}, {3.9, 9.864}}

```
(* THEN calculate dimensionless resistivity factors implicit in (C16) in UW CPTC 09-6R *)

InverseN = Inverse[Ne]
SpFactor = 1.0 / (Zeff InverseN[[1, 1]])
```

{{0.945154, -0.373692}, {-0.373692, 0.249128}}

0.406934

```
InverseNM = Inverse[Ne + Me]
ResFactor = 1.0 / (Zeff InverseNM[[1, 1]])
```

{{0.559323, -0.21309}, {-0.21309, 0.171993}}

0.687644

```
(* FINALLY calculate the neoclassical
  parallel resistivity in Eq. (C16) in UW CPTC 09-6R *)

etazero = me nue / (ne echarge^2)
(* reference, perpendicular resistivity in Ohm-m, defined before Eq. (2) in UW-CPTC 11-4 *)
```

$6.31327 \times 10^{-7}$

```
etaparnc = etazero ResFactor
(* DESIRED 2x2 MATRIX CALCULATION OF PARALLEL NEOCLASSICAL RESISTIVITY in Ohm-m,
Eq. (C16) in UW CPTC 09-6R *)
```

$4.34129 \times 10^{-7}$

```
(* AND THEN FINALLY CALCULATE KEY PALEOCLASSICAL PARAMETERS *)

Deta = etaparnc / muzero
(* magnetic field diffusivity parameter in m^2/s, Eq. (1) in UW-CPTC 11-4 *)
ScientificForm[%, 3]
```

0.345469

$3.45 \times 10^{-1}$

```
Mfactor = lambdae / (3.14159 Rzero q)
(* dimensionless collisionality factor M,
discussed in last paragraph in Section III of UW-CPTC 11-4 *)
ScientificForm[%, 3]
```

1.48585

1.49

```
(* COMPARISON to an adapted form for the neoclassical parallel resistivity
       given in Eq. (9) in UW-CPTC 11-4. The adaptations from original forms
       given in Eqs. (46)-(52) in Phys. Plasmas 12, 092512 (2005) are two-fold:
       1) the general nu_*e form used is given by Eq. (B12) in UW-CPTC 09-6R
    or Eq. (7) in UW-CPTC 11-
   4. It takes account of the large trapped particle fractions in pedestals,
  as prompted by J.M. Canik's analysis for NSTX pedestals where (~ 93%, f_t/f_c ~ 13), and
   2) the coefficient of the nu_*e factor in the denominator of Eq. (47) in
 Phys. Plasmas 12, 092512 (2005) is increased to 2 to be in accord with Eq. (B17) in UW-
 CPTC 09-6R plus a Pfirsch-
 Schluter regime factor is added -- both of which improve the fidelity of this formula *)

mueOVERnue = 1.2 (ft / fc) / ((1.0 + Sqrt[nustare] + 2.0 nustare) (1 + nue / omegate))
(* adapted from Eq. (47) in Phys. Plasmas 12,
092512 (2005) using factors in Eq. (B17) in UW-
 CPTC 09-6R or Eqs. (4)-(7) in UW-CPTC 11-4 *)
```

0.301998

```
etaApprox = etazero (SpFactor + mueOVERnue)
(* from Eq. (3) in UW-CPTC 11-4 *)
```

$4.47568 \times 10^{-7}$

```
RatioApprox = etaApprox  / etaparnc
(* ratio of this Approximation to this 2x2 matrix caclulation *)
```

1.03096

```
(* COMPARISON to neoclassical parallel resistivity often used in ONETWO,
which is based on S.P. Hirshman,
R.J. Hawryluk and B. Birge, Nuclear Fusion 17, 611 (1977),
as adapted in Eq. (7.41) in S.P. Hirshman and D.J. Sigmar, Nuclear Fusion 21, 1079 (1981),
and some other adaptations of Hirshman's formulas in the ONETWO neoresist subroutine
*)

cR = 0.56 Max[3.0 - Zeff, 0] / (Zeff (3.0  + Zeff))
(* this is same parameter as cra in ONETWO *)
```

0.0153846

```
xi = 0.58 + 0.20 Zeff
(* this is same parameter as zta in ONETWO *)
```

1.1

```
xft = 1.0 - (1.0 - epsilon)^2 / ((1.0 + 1.46 Sqrt[epsilon]) Sqrt[1.0 - epsilon^2])
```

0.759387

```
lnLambdaONETWO = 24.0 - Log[Sqrt[ne / 10^6] / Te]
```

14.6113

```
nustarHS = 1.414 (nue / Zeff) (lnLambdaONETWO / lnLambda) / (epsilon^1.5 omegate)
(* this is same parameter as xnusem in ONETWO *)
```

0.524896

```
ftad = xft / (1.0 + xi nustarHS)
```

0.481421

```
fftrap = ftad (1.0 + cR (1.0 - ftad))
```

0.485262

```
SpFactorHS = (1.0 + 1.198 Zeff + 0.222 Zeff^2) / (1.0 + 2.966 Zeff + 0.75 Zeff^2)
```

0.407465

```
etaONETWO = 100.0 etazero (lnLambdaONETWO / lnLambda) SpFactorHS / (1.0 - fftrap)
ScientificForm[%]
(* ONETWO resisivity in Ohm-cm;
linear interpolation in 98889 ONETWO outone file output gives
  5.16 x 10^{-5} x (2.6 / 2.83) ~ 4.74 x 10^{-5} for rho_n = 75.13 cm,
but the resistivity varies strongly with minor radius at this radial position *)
```

0.0000469651

$4.69651 \times 10^{-5}$

```
RatioONETWO = etaONETWO / (100.0 etaparnc)
  (* Ratio of calculation of eta in ONETWO to this 2x2 matrix calculation *)
```

1.08183

```
(* COMPARISON to the formula for the parallel neoclassical resistivity in Eq. (13a)
 in O. Sauter, C. Angioni and Y.R. Lin-Liu, Phys. Plasmas 6, 2834 (1999) *)
```

```
lnLambdaS = 31.3 - Log[Sqrt[ne] / Te]
(* Eq.(18d) in Sauter et al paper *)
ScientificForm[%, 3]
```

15.0036

$1.5 \times 10^{1}$

```
nustareS = 6.921 × 10^(-18) q Rzero ne Zeff lnLambdaS / (epsilon^1.5 Te^2)
(* Eq.(18b) in Sauter et al. paper *)
ScientificForm[%, 3]
```

1.39969

1.4

```
eN = 0.58 + 0.74 / (0.76 + Zeff)
(* Second part of Eq.(18a) in Sauter et al. paper *)
ScientificForm[%, 3]
```

0.800238

$8. \times 10^{-1}$

```
sigmaSp = 1.9012 × 10^4 Te^1.5 / (Zeff eN lnLambdaS)
(* First part of Eq. (18a) in Sauter et al. paper *)
ScientificForm[%, 3]
```

$4.02211 \times 10^{6}$

$4.02 \times 10^{6}$

```
X = ft / (1 + (0.55 - 0.1 ft) Sqrt[nustareS] + 0.45 fc nustareS / Zeff^1.5)
(* Eq. (13b) in Sauter et al paper *)
ScientificForm[%, 3]
```

0.481688

$4.82 \times 10^{-1}$

```
sigmancS = (1 - (1 + 0.36 / Zeff ) X + 0.59 X^2 / Zeff - 0.23 X^3 / Zeff) sigmaSp
(* Eq. (13a) in Sauter et al. paper *)
ScientificForm[%, 3]
```

$1.98846 \times 10^{6}$

$1.99 \times 10^{6}$

```
RatioSauter = 1 / (sigmancS etaparnc)
(* Ratio of Sauter et al. paper resistivity to this 2x2 matrix calculation *)
```

1.15842