Simulated flux-rope evolution during non-inductive startup in Pegasus

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Abstract. Magnetic flux ropes produced during non-inductive startup of the Pegasus Toroidal Experiment [Eidietis N W, et al., 2007 J. Fusion Energy 26 43] are simulated with nonlinear magnetohydrodynamic (MHD) and two-fluid plasma models. A single injector is represented by a localized source-density for magnetic helicity and thermal energy. Results show development of a hollow tokamak-like profile from a sequence of co-helicity merging events that release flux-rope rings from the driven flux rope. Accumulation of poloidal flux over many events redirects the driven flux rope so that its path traces a toroidal surface. Evidence of a quasi-separatrix layer is found from analysis of the squashing-degree parameter during a ring formation event in an MHD simulation. The layer bifurcates twice, once between non-reconnecting passes and again between reconnecting passes of the driven flux rope. Chaotic scattering during ring formation is also apparent from the distribution of field-line lengths. Correlation of flow-velocity and magnetic-field fluctuations, an MHD dynamo-like effect, concentrates symmetric poloidal flux during ring formation.

PACS codes: 52.30.Cv, 52.35Py, 52.35.Vd, 52.55Fa, 52.65Kj

Submitted to: Plasma Physics and Controlled Fusion

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1. Introduction

It is widely appreciated that magnetic flux ropes have important roles in solar flares and coronal mass ejections [1]. Laboratory facilities such as the Large Plasma Device (LAPD) [2], the Reconnection Scaling Experiment (RSX) [3], and experiments at Caltech [4] conduct studies of flux-ropes aimed toward understanding their basic dynamics. Flux ropes are also generated in some approaches to non-inductive current drive for magnetic confinement. Early examples include the Current Drive Experiment and the Continuous Current Tokamak [5,6], where voltage was applied to electron-emissive cathodes that intercepted vacuum magnetic field to drive plasma current and produce confining poloidal flux. A more recent example is the use of electrically biased washer-gun plasma sources for non-inductive startup in the Pegasus Toroidal Experiment [7]. In this work, we describe simulations of the nonlinear 3D evolution of flux ropes in Pegasus [8,9] and how the modeled dynamics and relaxation produce tokamak-like configurations. Our results demonstrate merging and magnetic reconnection, similar to flux-robe evolution in nature and in other experiments. Owing to the geometry of the driven helical current channel, the periodic release of flux-robe rings from co-helicity merging is a unique finding from our simulations [10].

The small aspect-ratio limit of the tokamak magnetic confinement configuration has magnetohydrodynamic (MHD) and confinement properties that are distinct from conventional configurations [11], and the spherical torus (ST) is a candidate for a fusion nuclear science test facility. The small cross-section of the central column presents a technological challenge in that it limits conventional inductive/transformer drive of toroidal plasma current. Developing suitable alternatives for plasma startup and profile control during sustenance is, therefore, an emphasis of the international ST research program.

Recognizing that the rotational transform required to confine charged particles in nominally symmetric toroidal systems implies linkage of poloidal and toroidal magnetic fluxes, current drive requires a source magnetic helicity, which tends to be well conserved globally in high-temperature plasma [12]. The approach tested on the Helicity Injected Torus (HIT [13] and HIT-II [14]), the Helicity Injected Spherical Torus (HIST [15]), and on the National Spherical Torus Experiment (NSTX [16]) uses toroidally symmetric electrode rings that intercept diverted or 'injector' poloidal flux. When the electrodes are electrically biased with respect to each other, there is net injection of helicity through the electrostatic surface contribution, \(-2\int \nabla \chi \cdot B \cdot dS\) where \(\chi\) is the electrostatic potential [17]. As shown in nonlinear simulations of straight [18] and ST configurations [19], macroscopic dynamics redistribute the current driven by the helicity injection as a form of magnetic relaxation. Recent experimentation on NSTX focuses on startup, and the transient coaxial helicity injection manipulates the induction of the expanding toroidal flux bubble to generate closed magnetic flux, similar to 2D representations of coronal mass ejection [20]. Recent simulation work finds evidence of Sweet-Parker type reconnection and relates the drop in toroidal-field pressure from the change in injector voltage with the onset of axisymmetric reconnection [21].

The development of washer-gun sources with robust and clean operation [22] sparked renewed interest in toroidally localized helicity sources for ST startup. Two geometrically different configurations have been investigated in Pegasus, which has a 1 m radius, 1.6 m high chamber, with the injectors mounted above the lower divertor plates in the first configuration and near the outboard midplane in the second configuration. Experiments with the first configuration established the formation of a diffuse,
tokamak-like discharge from localized sources, and it distinguished operational regimes for distinct-filament, sheet, and diffuse discharges, as inferred from visible-light images [8]. Another significant result is that magnetic relaxation and multiplication of plasma current relative to the vacuum winding become significant when the self-induced magnetic field exceeds the vacuum vertical field on the surface of the central column. Results with the second configuration show that increasing the vertical field helps maintain radial force-balance and adds poloidal-field (PF) induction as the plasma expands toward the central column [9]. In addition, the plasma transitions to a quiescent state suitable for handoff to transformer or other current drive after the non-inductive injection is removed.

Focusing on the basic flux-rope dynamics of non-inductive current drive in Pegasus, our simulations consider the divertor gun configuration without PF induction. Physical parameters and numerical modeling are described in the following section. Section 3 describes different phases of the simulated evolution and reviews phenomenological results presented in Ref. [10]. Section 4 considers topology evolution during merger and reconnection of the simulated flux rope. In Section 5, we consider the dynamo-like correlation of asymmetric fields that induce changes to the symmetric profile. Conclusions and open questions are discussed in the final section.

2. Parameters and modeling
The helicity injectors used in Pegasus contain arrangements of electrode and insulator washers that are stacked to allow feed-through gas to be ionized and emitted through an aperture of only 1-2 cm in diameter [22]. These miniature washer-guns necessarily release some neutral gas prior to forming the internal discharge. This prefills the cylindrical vacuum tank with 1-5×10⁵ Torr of H₂ or D₂ [23]. Application of voltage across the internal electrodes ionizes the flowing gas, establishing an arc discharge within the injector assembly. Electron number density and temperature at the aperture for the first injectors used in Pegasus were measured to be n = 1-3×10²⁰ m⁻³ and Tₑ = 10-20 eV, respectively [22]. The warm plasma is emitted along externally supplied, vacuum toroidal and vertical components of magnetic field (B_vac) and establishes an electrically conducting path. Current filaments, i.e. flux ropes, are formed when the injector’s aperture is biased as a cathode relative to an anode plate mounted near the top of the Pegasus vacuum vessel or with respect to the vessel, itself.

The initial diameter of an injected current filament is comparable to the diameter of the aperture, which is more than two orders of magnitude smaller than the path-length along the vacuum field from the injector to the anode. The different scales produce the characteristic flux-rope geometry. They also stress spatial resolution requirements in numerical simulations. Thus, our modeling simplifies several aspects of the Pegasus experiments. We do not attempt to model the injectors themselves but include a localized volumetric source of helicity, +2∫E_{inj}·B dVol with E_{inj} = ηµ₀⁻¹λ_{inj} B and η being the electrical resistivity. The source is only applied in a small region near the location where the guns are mounted in the experiment,

\[ \lambda_{inj}(x,t) = \lambda_0(t) \exp \left[ - \frac{\left( R - R_{inj} \right)}{w_{inj}} \left( \frac{Z - Z_{inj}(\phi)}{w_{inj}} \right)^2 - \left( \frac{n_{inj} \phi}{\pi} \right)^2 \right], \]  

with a poloidal 'width' w_{inj} = 4 cm, radial location R_{inj} = 35 cm, axial positioning that follows the vacuum field in the toroidal angle \( \phi \) with Z_{inj}(0) = −70 cm (relative to the Pegasus midplane), and
toroidal scale limited with $n_{inj} = 4$. This enlarges the cross-section of the flux rope relative to the experiment but still allows a clear separation of spatial scales, as shown in Fig. 1 of Ref. [10]. The vertical component of $B_{vac}$ in our computations is 3.7 mT and the toroidal component at $R_{inj}$ is 20 mT, hence the geometric winding of $\Delta z B_{\phi} / 2\pi R_{inj} B_c = 4$. The amplitude $\lambda_0(t)$ is increased linearly in time at a rate of $200 \text{ m}^{-1}\text{s}$, allowing more gradual current development than the sudden application of injector voltage in the experiment. The experimental campaign with the first geometric configuration considered both single- and two-injector operation, while our simulations have just one source region.

We model plasma throughout the Pegasus chamber with two-temperature resistive-MHD and low-frequency two-fluid systems of equations. The two-fluid system is

$$\frac{\partial n}{\partial t} = -\nabla \cdot \left( n \mathbf{V} - D_n \nabla n + D_h \nabla^2 n \right)$$

$$+ n \left( \frac{\partial}{\partial t} \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \left( n m \mathbf{W} \right)$$

$$- \frac{2n}{3} \left( \frac{\partial}{\partial t} T_e + \mathbf{V} \cdot \nabla T_e \right) = -n T_e \nabla \cdot \mathbf{V}_e + \nabla \cdot \left[ \left( \kappa_{le} - \kappa_{le} \right) \mathbf{b} \mathbf{b} + \kappa_{le} \mathbf{I} \right] \nabla T_e + n \sigma (T_i - T_e) + \eta \mathbf{J} \times \mathbf{B}$$

$$- \frac{2n}{3} \left( \frac{\partial}{\partial t} T_i + \mathbf{V} \cdot \nabla T_i \right) = -n T_i \nabla \cdot \mathbf{V}_i + \nabla \cdot \left[ \left( \kappa_{li} - \kappa_{li} \right) \mathbf{b} \mathbf{b} + \kappa_{li} \mathbf{I} \right] \nabla T_i + n \sigma (T_e - T_i) + Q_i$$

$$\frac{\partial}{\partial t} \mathbf{B} = -\nabla \times \left[ \eta \left( \frac{\lambda_{inj}}{\mu_0} \mathbf{B} \right) - \mathbf{V} \times \mathbf{B} + \frac{1}{ne} \left( \mathbf{J} \times \mathbf{B} - \nabla p_e \right) \right],$$

where $\mathbf{V}$ is the center-of-mass flow velocity, $p$ is the sum of electron and ion pressures, temperatures are in units of energy, and $\mathbf{J} = \mu_0^{-1} \mathbf{V} \times \mathbf{B}$ is the current density. Transport coefficients include $\eta(T_e) \sim T_e^{-3/2}$ with $\mu_0^{-1} \eta(1 \text{ eV}) = 100 \text{ m}^2/\text{s}$, anisotropic thermal conduction with magnetization effects from Braginskii [24,10], and thermal equilibration with $\sigma = 3m_e/\tau_e m_i$ with temperature- and density-dependent collision time $\tau_e$. We include a phenomenological isotropic viscous stress with $\mathbf{W}$ being the traceless rate-of-strain tensor and $\nu$ set to 10 m$^2$/s; a corresponding heating is not retained in the ion temperature equation. The density evolution also has diffusive contributions to represent anomalous effects with $D_n = 1 \text{ m}^2/\text{s}$ and a numerical hyper-diffusivity of $D_h = 0.1 \text{ m}^4/\text{s}$. Equation (2e) is Faraday's law with the two-fluid Ohm's law and the helicity source. The Hall and $\nabla p_e$ terms are not included in the resistive-MHD computations; otherwise, the system is the same as (2a-e).

We solve Eqs. (2a-e) for initial-value problems using the Non-Ideal Magnetohydrodynamics with Rotation, Open Discussion (NIMROD) code [25]. The MHD computations use a semi-implicit temporal advance, and the two-fluid computations use an implicit leapfrog method [26]. With the relaxed range of spatial scales, a 24×24 mesh of biquartic elements for the $R-Z$ plane with Fourier components 0 ≤ $n$ ≤ 10 for the toroidal direction has proven sufficiently accurate. The computations model non-inductive startup beginning with vacuum-field conditions and $T_0 = 0.25 \text{ eV}$ 'plasma' fluid with $n = 1 \times 10^{19} \text{ m}^{-3}$ throughout the chamber. The heat-source density $Q_i$ has the spatial distribution of $\lambda_{inj}$ shown in Eq. (1) to model plasma sourcing in Pegasus. Anisotropic thermal conduction of magnetized electrons forms an
electrically conducting helical path that extends from the source region to the axial ends of the vessel. Completing the circuit requires sufficient electrical conductivity near these surfaces. In the absence of sheath and plasma-surface modeling, we apply a thermally insulating boundary condition when solving (2c-d) but include the linear relaxation term, \(-\left(2n\alpha_T/3\right)(T-T_0)\) with \(\alpha_T = 10^4 \text{ s}^{-1}\) for ions and \(\alpha_T = 10^5 \text{ s}^{-1}\) for electrons, along the surface of the domain. Also, the Pegasus vessel allows magnetic diffusion on the time-scale of the discharges. We have found that modeling the chamber as a conducting wall leads to an unphysical outward shift of toroidal flux that impedes axial current. Relaxing the toroidal component of (2e), \(-\alpha_\eta\left(B_\phi - B_{\text{vac}_\phi}\right)\) with \(\alpha_\eta = 10^7 \text{ s}^{-1}\) along the inboard and outboard surfaces, avoids the unphysical shift and represents a resistive-wall in a very approximate way.

3. Phases of flux-rope evolution

From Faraday's law, application of the localized \(E_{\text{inj}}\) directly induces a ring of magnetic flux around the source region. Super-posing this ring on the vacuum field can be visualized as enclosing a piece of candy with an axially striped (magnetic field-lines) wrapper by twisting it around the candy center with the ends fixed. The twist excites propagation of torsional Alfvén waves, initially surrounded by reversed current, similar to the waves described in Ref. [2]. The unsupported reversed current quickly diffuses through the cold regions surrounding our modeled helical channel, leaving a flux rope with net current. Figure 1 of Ref. [10] shows an isosurface of the parallel-current parameter, \(\lambda = \mu_0 j_\parallel / B\), when the simulated filament still traces the vacuum magnetic field.

With one source region, there is only one driven flux rope, but the vacuum field needed to generate a tokamak-like configuration directs the rope along several helical passes [23]. The currents crossing any constant-\(\phi\) plane are necessarily parallel, so adjacent passes attract each other. The distribution resembles the classic description of island-coalescence instability [27], but the loose-solenoid equilibrium is neither sinusoidal nor periodic, and the source ties the structure near one end. Comparison with analytical [28,29] and other numerical studies [30] where ropes do not carry net current is also imprecise. When the magnetic field associated with the current filament reaches the magnitude of the vertical field, the attractive forces excite vertical oscillations in the helical channel.

As the rate of injection is increased, the influence of the flux-rope field extends over greater distances measured from the flux-rope axis. It reduces the vertical field inboard of the rope and reinforces it on the outboard side. Interaction among adjacent passes intensifies over time until they collide and merge via magnetic reconnection. As shown in Fig. 1, the merger geometry is nearly parallel, and adjacent passes are oriented in the co-helicity sense. The reconnection site appears similar to the RR0 geometry of Ref. [30], but two of the rope ends curve away vertically to the ends of the current path. Reconnection along the rope is, therefore, restricted, as in RSX [31] and LAPD [32]. Figure 2 of Ref. [10] shows that the cross-section of the reversed current density that is associated with reconnection has the characteristic S-shape for flux-rope merger, also reported in Refs. [33,34]. In the Pegasus simulations, the localized merger releases a ring of net current that surrounds the central column. The periodic creation and spreading of flux-rope rings accumulate net axisymmetric poloidal flux as part of the magnetic relaxation process [10]. An early numerical study of the interaction of flux-ropes also finds conditions that produce a flux-rope ring [35]. In that case, the two distinct ropes carry net current but are driven by field-lines tied
to foot-point motion, and a ring is produced in the attracting case with opposite helicity. Closed loops in
the study of ropes without net current also form in cases of opposite helicity only [30].

Laboratory results from Pegasus show that the accumulation of toroidal plasma current accelerates
when magnetic field from injection reverses the applied vertical field on the inboard side of the chamber
[8]. Our simulations find that large-scale reversal also leads to a hollow toroidal current path [10]. Early
in this phase, rings continue to form on the outboard side of the magnetic structure. Later, the flux rope
traces a toroidal surface, as evident from Fig. 2a, and merger occurs on the inboard side, which releases
rings within the toroidal surface. Camera images of Pegasus discharges late in the injection show a
diffuse source of light from a toroidally shaped region and not a clear channel [8]. However, both
simulations and experiment find magnetic fluctuations of approximately 5% amplitude and 20 kHz
frequency from magnetic pickup coils near the outboard midplane during this phase. A rope moving
vertically past a midplane probe imparts a signal at twice the frequency of its oscillation, and in the
simulation results, it is clear that the 20 kHz signal is associated with dynamics excited by flux-rope
merger. When injection is terminated in a simulation, the flux-rope structure diffuses rapidly, leaving a
relatively symmetric toroidal configuration, shown in Fig. 2b, with closed nested magnetic flux surfaces
over a significant fraction of the domain cross-section.

The potential importance of effects beyond the scope of resistive MHD can be inferred from the
plasma parameters and the flux-rope diameter. With $n ≈ 1×10^{19}$ m$^{-3}$, the skin-depth of the ion (D)
species is 10 cm, which is comparable to the flux-rope diameter in our simulations. At the radius of the
source, the sound-gyroradius $\rho_s = c_s / \Omega_i$ is approximately 3 cm, where $c_s$ is the ion-acoustic speed and
$\Omega_i$ is the ion cyclotron frequency. With the low values of $T_e ≈ 5$ eV observed during early ring
formation and reconnection occurring over $\tau_{rec} ≈ 50$ $\mu$s, the resistive skin-depth, $\sqrt{\mu_0 \eta \tau_{rec}}$ is
approximately 2 cm. Thus, two-fluid effects can be expected during reconnection [36]. While there are
quantitative differences between the results of our MHD and two-fluid computations, the evolution
proceeds in the same way, qualitatively. The absence of significant two-fluid effects may be due to the
fact that the current remains aligned with the magnetic field, so the Hall contribution to the electric field
remains small. Quantitative differences associated with poloidal flux development are described in Sect.
5.

4. Topology evolution
The formation of a flux-rope ring represents a topological change for magnetic field-lines. Reference [35]
shows that formation of a ring can also involve chaotic scattering and fractal behavior for foot-point-
driven flux ropes with parallel current and opposite helicity. Here, we consider whether the co-helicity
orientation of ring formation in Pegasus has similar features. This orientation is illustrated by the
magnetic field-line traces shown in Fig. 3, where parallel and anti-parallel trajectories are launched from a
grid of 13 points that cover the source region in the MHD simulation. The two times shown in the figure
represent a small fraction of the ring formation in Fig. 1. At the earlier time of Figs. 3a-b, the 13 traces
are topologically unchanged from conditions before the start of this reconnection event. Shortly
thereafter, some of the traces make more than one pass around the ring region before progressing to the
top of the domain, as evident in Figs. 3c-d. As reconnection continues, the ring separates from the driven
flux rope.
Identification of quasi-separatrix layers (QSLs) of a magnetic field distribution helps locate regions where reconnection occurs in open field-line systems [37]. For example, Ref. [34] identifies QSLs in extensive probe data from LAPD to confirm and locate reconnection among flux ropes. The analysis is readily applied to our simulation data, where we trace field-lines from their entrance along the bottom of the domain to their exit at the top. We use a 2000x60 (radial×azimuthal) mesh of launch points to provide a discrete representation of the field-line mapping functions, \(x_i(x_0,y_0)\) and \(y_i(x_0,y_0)\), where \(x\) and \(y\) are Cartesian coordinates in the constant-Z planes of the bottom ("0" subscript) and top ("1") surfaces. A QSL can be identified by large values of the squashing degree \(Q\) [38]. In cases such as our simulations, where \(\mathbf{B} \cdot \mathbf{n}\) is uniform and has the same absolute value on the entry and exit surfaces, \(Q\) is the same as the square of the norm of the mapping,

\[
Q = \left( \frac{\partial x_1}{\partial x_0} \right)^2 + \left( \frac{\partial y_1}{\partial x_0} \right)^2 + \left( \frac{\partial x_1}{\partial y_0} \right)^2 + \left( \frac{\partial y_1}{\partial y_0} \right)^2 .
\]

(3)

Contours of \(\ln(Q)\), computed from finite differences of the discrete mapping information from the MHD simulation results at \(t = 2.63\) ms, are plotted with respect to launch position in Fig. 4a. The contour levels are limited to highlight large values of \(Q\), and launch points have \(R_0 > 0.15\) m to avoid the large values of \(Q\) near the central column from the externally sourced vacuum field.¹

During our simulated flux-rope merging, large \(Q\)-values appear as rolled surfaces, as indicated by their projection onto the \(Z_0\) plane, Fig. 4a. To determine whether they indicate the formation of a QSL that is related to magnetic reconnection, we consider where the large-\(Q\) trajectories intercept the constant-\(\phi\) plane that has the greatest reversed current density. Figure 4b shows this information with the Poincaré plot for trajectories with \(\ln(Q) > 13.2\) and \(R_0 > 0.23\) m overlaid on contours of constant-\(\lambda\) for the \(\phi = 3\pi/2\) plane. As these trajectories progress upward from the bottom surface, they wrap around the inboard side of the current channel and then bifurcate at \(Z = -0.55\) m. Some continue around the inboard side of the next pass of the current channel, while others continue around its outboard side. Those on the inboard side bifurcate again at \(Z = -0.2\) m, which is the site of reconnection, as indicated by large negative values of \(\lambda\). This second bifurcation is consistent with the relation between QSLs and magnetic reconnection presented in Refs. [37,38].

To understand the first bifurcation, we draw an analogy with periodic systems. Magnetic island chains from tearing can form in magnetically sheared periodic systems along a resonant surface, where \(\mathbf{B} \cdot \mathbf{k} = 0\) with \(\mathbf{k}\) being the helical wavenumber vector. If the island chain is subject to coalescence instability, the distance between some adjacent \(O\)-points decreases, while the distance between others increases. For small-amplitude, helically symmetric perturbations, the \(X\)-points of the magnetic separatrix remain on the resonant surface, and both \(X\)- and \(O\)-points can be identified by nulls of the helical field \(\mathbf{B}_h = B_k \hat{R} + \mathbf{B} \cdot \hat{\mathbf{k}}\). The analogy considers the cylindrical surface \(R = R_{ni}\) in Pegasus to be similar to the resonant surface in a periodic system. We choose \(\mathbf{k} = 2\pi n_z \Delta z^{-1} \hat{z} - R_{ni}^{-1} \hat{\phi}\) with \(n_z = 4\) for the geometric winding of the vacuum field at the injector location. In Fig. 4c, we plot the magnitude of this

¹ Values of \(B_z\) and \(I = RB_\phi\) are uniform in the vacuum-field, and the squashing degree has the simple 1D dependence, \(Q = 2 + 4 |\Delta \phi|^2(R)\), where \(\Delta \phi(R) = I \Delta z / R^2 B_z\). For the parameters of our simulation, the vacuum-field value of \(Q\) along the central column, \(R = 5\) cm, is nearly \(6 \times 10^6\).
with the same magnetic punctures from 4b. The overlay shows that the first bifurcation is near the minimum in $|B_n|$ between the lowest two passes of the flux rope, and the second bifurcation is near the minimum in $|B_n|$ between the second and third passes, where reconnection is occurring. This confirms that the first bifurcation also represents an X-point like behavior, although it is not from magnetic reconnection.

Large $Q$-values can be expected more generally from chaotic scattering of trajectories around the forming ring, and chaotic scattering is evident from the plots of field-line length in Fig. 5. Data for this figure is determined from 50,000 trajectories launched from the line segment $10 \text{ cm} \leq R \leq 50 \text{ cm}$ at $\phi = -\pi / 4$ and $Z = Z_0$. Regions where long trajectories rise above a background with the longest values appearing at the edges of each region, i.e. castle-like structures, appear across different spatial scales in $R$. These structures indicate chaotic scattering around magnetic structures and have been identified previously in the flux-rope simulations of Ref. [35] and in weakly driven conditions for spheromaks in Ref. [39].

5. Poloidal flux development

In Pegasus, biasing the injector with respect to the vessel after emitting plasma along the helical vacuum field is like applying DC voltage to a loose solenoid, where induction of flux threading and surrounding the windings is analogous to the experiment's generation of poloidal flux. Unlike a common solenoid, however, plasma flux-rope dynamics change the shape and winding of the current path while the poloidal flux builds. The helicity source in our simulations induces poloidal flux directly, $\int E_{\text{inj}} \cdot \phi R d\phi \neq 0$, but wave propagation and macroscopic activity are needed to develop a tokamak-like distribution. The simulation results indicate a multi-step process. The source increasingly twists the rope and torsional waves propagate the twist on a rapid timescale; the helical path is 8.9 m, and the local Alfvén speed is $9.8 \times 10^4 \text{ m/s}$. As the island-coalescence instability develops, a helical section of the rope rotates to approximately align with a constant-$Z$ plane as a result of attracting vertical displacements. The rope is only constrained by the source region, but on the time-scale of the vertical displacements, the endpoints are essentially unchanged. The rotation converts helical flux into a concentration of poloidal flux in the vicinity of the forming ring. When the ring is released through reconnection, resistive diffusion spreads the ring. The shortened driven channel is again subject to the twisting that builds helical flux, increasing its length and number of windings in the process.

In reversed-field pinches and spheromaks, flux conversion from multi-helicity dynamics plays a large role in forming the configuration. The effects can be assessed from dynamo-like correlations of magnetic fluctuations with fluctuations of flow-velocity and current density, i.e. the $-\langle \mathbf{V} \times \mathbf{B} \rangle$ MHD and $(ne)^{-1}\langle \mathbf{J} \times \mathbf{B} \rangle$ Hall electric fields [40], where $\langle \rangle$ indicates a spatial average and $\sim$ signifies the asymmetric components of a field. The analysis can be applied to the flux-rope dynamics in our simulations. In Fig. 6a, we show the $-\langle \mathbf{V} \times \mathbf{B} \rangle_{\phi}$ correlation from the MHD computation. Positive values near the helicity source oppose the generation of flux near the source, indicating transfer of energy into fluctuations, as $-\langle \mathbf{E} \rangle \cdot \langle \mathbf{J} \rangle$ in Poynting's theorem is negative [41]. Where the symmetric ring is forming, there is a region with negative values. The negative values coincide with the symmetric ring current shown in Fig. 6b and induce the poloidal flux concentration evident in Fig. 6a. The toroidally asymmetric
ends of the flux rope that lead away from the forming ring contribute to $-\langle \vec{V} \times \vec{B} \rangle_\phi$ as they feed asymmetric field into the reconnection site where it becomes symmetric.

Both MHD and Hall correlations are active during ring formation in our two-fluid computation. Figure 7a shows that the MHD field induces poloidal field on the scale of the ring. The Hall field, shown in Fig. 7b, is of comparable magnitude, but it changes sign over the smaller spatial scale of the reconnection current, indicated by the contours of $\lambda$ from the $\phi = 0$ plane. Thus, the fluctuation-induced Hall effect may contribute more to shaping the ring and less to transferring net energy than the MHD effect.

Conclusions
The formation of flux-rope rings, as predicted by our simulations, is a plausible mechanism for developing tokamak-like profiles from non-inductive current drive in Pegasus. As noted previously, ring formation has been observed in other studies. However, generation from co-helicity merging may be unique to the tokamak startup configuration. The merging magnetic field in two parallel-current co-helicity tubes becomes increasingly parallel as reconnection proceeds into the interior of the tubes, which limits reconnection [30]. In the tokamak startup configuration, loss of current drive in the ring aids separation of the ring from the driven helical channel as reconnection proceeds [10].

Results from our squashing-degree analysis of ring formation in Sect. 4 show the influence of the helical flux-tube geometry. Some of the field-line trajectories with large $Q$-values pass through the reconnection site, but the QSL structure bifurcates between the first two passes of the flux rope before bifurcating again between the second and third passes, where reconnection occurs. Both represent separatrix-like behavior, hence the QSL label is appropriate. The chaotic scattering of trajectories, confirmed by the fractal nature of field-line lengths, also produces large $Q$-values as the ring develops. An aspect of the dynamics that is relatively straightforward is the origin of dynamo-like electric field from the correlation of flow-velocity and magnetic-field fluctuations over the toroidal angle. The asymmetries represent the ends of the flux tube as they are fed into the ring, and the emf resulting from the correlation is consistent with the resulting concentration of symmetric poloidal flux.

While flux ropes are evident in camera images of visible light from Pegasus discharges dominated by the vacuum field, they are not distinguishable when tokamak-like discharges are produced. This may be a matter of temporal resolution for the imaging or that magnetic structures and the source of light are not well correlated in the diffuse-discharge regime. New probe diagnostics are being developed for Pegasus to better measure small spatial scales in the magnetic field. The importance of effects not included in our fluid modeling, such as energetic particles, turbulent transport, and interactions with neutrals, also needs greater consideration. Finally, operation with more than one plasma source, hence the generation of more than one flux rope, may lead to other topological changes like braiding, in addition to ring formation.

Acknowledgments
We thank Prof. Raymond Fonck and members of the Pegasus team for valuable discussions on the experiments. Dr. Michael Bongard also provided helpful feedback on our manuscript. This work is funded through U.S. Department of Energy Grant DE-FC02-05ER54813 as part of the Plasma Science and Innovation Center. Some of our Pegasus computations have been performed at the National Energy
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Figure 1. Isosurfaces of $\lambda$ (m$^{-1}$) at times 2.63, 2.66, and 2.69 ms (left, center, and right) when $I_p \approx 6.4$ kA in the MHD computation, prior to large-scale poloidal-field reversal. The positive-valued (red) isosurface shows current flowing from the source, and the negative-valued (blue) isosurface shows reversed current associated with reconnection.
Figure 2. Isosurfaces of $\lambda$ (a) late in the injection phase of the MHD computation when $I_p = 46$ kA and (b) with $\lambda > 1.5$ m$^{-1}$ contours, 2 ms after injection is stopped.
Figure 3. Field-line traces from a grid of 13 positions covering the source region in the MHD simulation. The traces in (a) and in the zoomed image (b) are from 2.649 ms into the simulation. Those in (c) and (d) are just 5 μs later, and the darkened trace accentuates a trajectory that makes more than one pass through the forming ring.
Figure 4. Values and traces from squashing-degree ($Q$) analysis of the MHD results at $t = 2.63$ ms. Contours of $\ln(Q)$ (for $\ln(Q) > 8$) are plotted in (a) with respect to launch position on the bottom surface. Contours in (b) are the parallel-current parameter $\lambda$ for the $\phi = 3\pi/2$ plane, and contours in (c) are the magnitude of $B_h$ at $\phi = 3\pi/2$. Magnetic field-line punctures of the $\phi = 3\pi/2$ plane, for traces having $\ln(Q) > 13.2$ and $R_0 > 0.23$ m, are overlaid in (b) and (c).
Figure 5. Field-line length as a function of $R_0$ at $t = 2.65$ ms in the MHD computation (left) and with an expanded scale (right). The traces are launched from $Z_0$ in the $\phi = -\pi / 4$ plane.
Figure 6. Poloidal-flux development at $t = 2.65$ ms, during the early ring formation event of Fig. 1. Contours of emf induced by MHD fluctuations, $-2\pi R \langle \mathbf{V} \times \mathbf{B} \rangle$, are shown in color together with poloidal flux contours in (a). Contours of toroidally averaged $J_\phi$ are shown in (b).
Figure 7. Flux development during a ring formation event in the two-fluid computation. Contours of $-2\pi R \langle \mathbf{V} \times \mathbf{B} \rangle_{\phi}$ are shown in color together with positive (solid) and negative (dashed) contours of $\langle J_{\phi} \rangle$ in (a). Contours of $2\pi R (ne)^{-1} \langle \mathbf{J} \times \mathbf{B} \rangle_{\phi}$ are shown in color with positive (solid) and negative (dashed) contours of $\lambda(\phi = 0)$.