Flow Damping Due to a Model of Fluctuation Induced Viscosity in DEBS

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The global viscous damping rate in MST has been studied experimentally[1]. In these experiments a biased electrode was applied to a steady RFP plasma with $\Theta \sim 1.8$. This causes a radial electric field E_r, which in turn induces an increased toroidal velocity in the discharge that reaches a steady value. When the electrode is removed, the toroidal flow is damped with a characteristic time $\tau_{sd} \sim 0.5$ msec. The effective viscous diffusion (flow damping) coefficient was then estimated as $\nu_\perp \sim (\Delta r)^2/\tau_{sd} \simeq 50\text{m}^2/\text{sec}$, where $\Delta r = 0.37$ m is the radial extent of the core region. This estimate is several orders of magnitude larger than that implied by the standard collisional perpendicular viscosity, so that the viscosity in MST is anomalous. It is stated in Ref. [1] that $\nu_\perp$ is consistent with a theoretically derived viscosity driven by magnetic fluctuations[2],

$$\nu_\perp \simeq \left(\frac{\delta B}{B_0}\right)^2 LC_s \ , \tag{1}$$

where $\delta B/B_0$ is the relative amplitude of the magnetic fluctuations, C_s is the parallel sound speed, and L is the parallel correlation length. In Ref [2], the latter is taken as as $L \sim qR$, where q is the safety factor and R is the major radius.

We have implemented a viscosity coefficient based in Equation (1) in the DEBS code[3] and simulated the process of spin-up and flow damping in a cylindrical model of an RFP. In this case, an axial (V_z) flow is induced by modifying the momentum equation for the $(0,0)$ flow as

$$\frac{\partial V_z}{\partial t} = \ldots + \nu_z \left[V_{z0}(r) - V_z\right] \ , \tag{2}$$

where $V_{z0}(r)$ is a specified velocity profile of the form

$$V_{z0}(r) = V_{za}\left[1 - \left(\frac{r}{a}\right)^a\right]^b \ , \tag{3}$$

V_{za} is an amplitude normalized to the Alfvén velocity, a and b are positive integers, and ν_z is a drag coefficient that forces the mean flow to evolve toward the desired profile $V_{z0}(r)$. We first set $\nu_z = 0$, apply a voltage to keep Θ approximately constant, and allow a quasi-steady state, sawtoothing RFP state to emerge. We then apply $\nu_z > 0$ for some time until another quasi-steady state with mean axial flow V_z is reached. When we again set $\nu_z = 0$, the mean flow is damped by the viscosity, Equation (1), and the damping rate can be measured and compared with that of Ref. [1].

In DEBS, time is normalized to the resistive diffusion time τ_R, and the normalized viscous coefficient is $\nu = \tau_R/\tau_\nu \equiv P_M$, the magnetic Prandtl number. We apply this normalization to Equation (1), and use $C_s = \sqrt{\Gamma\beta_0/2}$ and $L = \alpha_\nu q_0 R$, where $\alpha_\nu \geq 0$ is a constant of proportionality to be specified. Here we have taken $q \sim q_0 = q(0)$ to be constant, since Equation (1) was derived for tokamaks where $q > 0$ everywhere. Then using the RFP relationship $Rq_0 = a/\Theta$, where $\Theta = B_\theta(a)/ < B_z >$ is the pinch ratio, the non-dimensional viscosity is

$$\nu = \alpha_\nu \sqrt{\frac{\Gamma\beta_0}{2}}\frac{S}{\Theta}\left(\frac{\delta B}{B_0}\right)^2 \ , \tag{4}$$

where $S = \tau_R/\tau_A$ is the Lundquist number. The fluctuations

$$\left(\frac{\delta B}{B_0}\right)^2 = \frac{1}{B_0(r)^2}\sum_{m,k} B_r(r,m,k)B_r^*(r,m,k) \tag{5}$$

are computed in DEBS as a function or radius and time; everything else is constant. For the force-free model to be used here, β_0 is a specified characteristic value of $\beta = 2\mu_0 p_0/B_0^2$. (For finite-β calculations with pressure evolution, both β_0 and S would also be functions of radius and time.) The viscosity therefore varies with both radius and time as $\delta B/B_0$ evolves.

We have used the parameters $a = 0.5$ m, $R/a = 3$, $B_0 = 0.2$ T, and $n = 8 \times 10^{18}$ /m^3. These are close to the parameters in the experiments[1]. We have assumed a Deuterium plasma and apply a voltage to maintain $\Theta \simeq 1.8$. These lead to $V_A = 1.09 \times 10^6$ m/sec and $\tau_A = a/V_A = 4.58 \times 10^{-7}$ sec. The spatial resolution is 50 radial mesh points, 341 axial modes ($-170 \leq n \leq 170$) and 6 poloidal modes ($0 \geq m \leq 5$). The maximum time step is $0.5\tau_A$. We have taken $S = 10^5$, which is likely smaller than occurs in the experiments but is as large as is computationally practical for this resolution. Then the resistive diffusion time is $\tau_R = 4.58 \times 10^{-2}$

sec. (We have found that $S = 10^6$ becomes numerically unstable, and such runs likely require finer radial resolution.) We take the constant of proportionality in Equation (4) to be $\alpha_\nu = 15.7$, which is consistent with the assumption $L = 2\pi a^2$ for the parallel correlation length used in Ref. [1], [4]. The "background" viscosity is taken to be neglegable (rhocon $= 10^{-3}$ and rcell $= 10^{20}$, for DEBS afficianodos); the viscosity is entirely due to the fluctuations, as given by Equations (4) and (5). DEBS is then run with these parameters until a quasi-steady RFP dynamo with $F = B_z(a)/ < B_z > \sim -0.17$ emerges.

For the spin-up phase, in Equation (3) we take $\nu_z = \tau_R/\tau_s = 30.564$ and $V_{za}/V_A = 2.75 \times 10^{-2}$. These correspond to the reported experimental values $\tau_s = 1.5$ msec and $V_{za} = 30$ km/sec. In Equation (3) we also take $a = 30$ and $b = 2$, which results in a very flat flow profile. Together these lead to a second quasi-steady state with axial flow of $V_z \sim 15.3$ km/sec that is maintained for ~ 5 msec. The value of the flow is somewhat lower than the target V_{za} because the term highlighted in Equation (2) is partially opposed by the axial viscous force arising from Equation (4). We then set $\nu_z = 0$, allow the flow to decay viscously due to Equation (4) and measure the decay time.

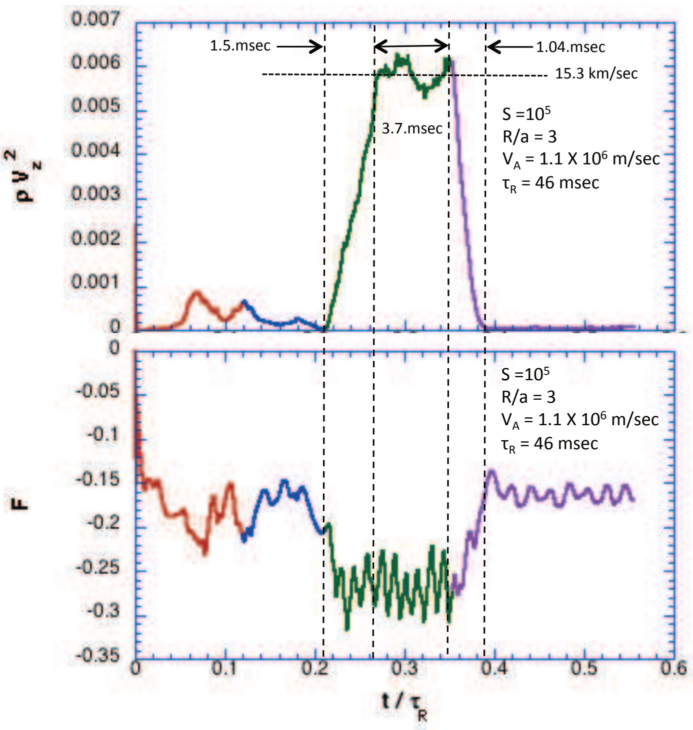

Figure 1: Axial kinetic energy ρV_z^2 (top) and the field reversal parameter F (bottom) as functions of time before, during, and after the application of the pulse.

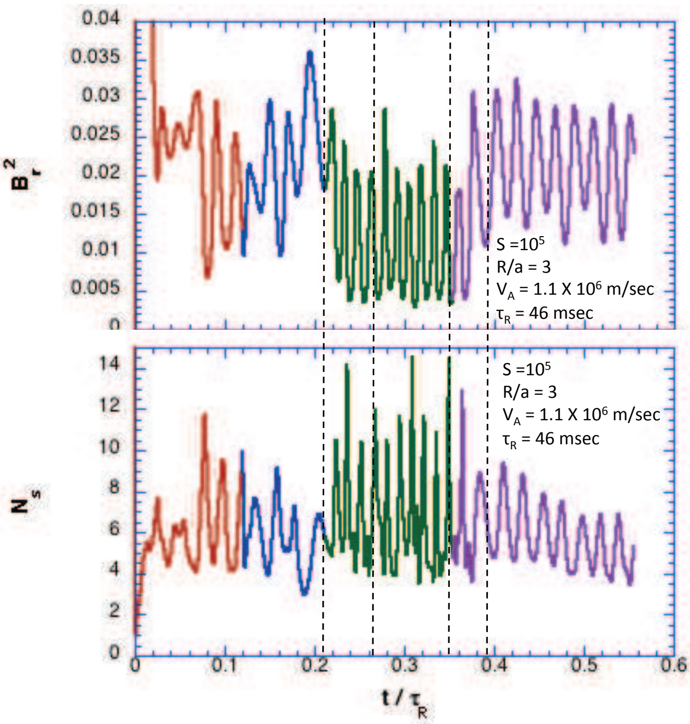

Figure 2: Radial magnetic energy B_r^2 (top) and N_s, the the number of $m = 1$ modes participating in the dynamo (bottom), as functions of time before, during, and after the application of the pulse.

The results are summarized in Figures 1 and 2. Figure 1 (top) shows the non-dimensional axial kinetic energy ρV_z^2 as a function of t/τ_R. After a dynamo state is established, spin-up is initiated at $t/\tau_R \simeq 0.21$. A second quasi-steady state is achieved in $\sim$ 1.5 msec and maintained for another $\sim$ 3.7 msec. This is shorter than the sustained state reported in Ref. [1] ($\sim$ 10 msec), but seems sufficient for our purposed. The axial drive is terminated at $\sim 0.34 t/\tau_R$, and the axial velocity decays exponentially as e^{-44t/τ_R}. This corresponds to a viscous decay time of $\tau_{sd} = 1.04$ msec, and an effective viscous diffusion coefficient of

$$\nu \equiv \frac{a^2}{\tau_{sp}} = 120 \ m^2/sec \ . \tag{6}$$

This is to be compared with the value $\nu = 50$ m^2/sec estimated in Ref. [1], which used the core radius $a_{core} = 0.37$ m instead of the minor radius $a = 0.5$ m. Using a_{core} in Equation (6) leads to $\nu = 66$ m^2/sec. However, the axial flow profile in the computations is likely much flatter than in the experiment, and using the minor radius seems more appropriate. This leads to an estimate of the viscous diffusion coefficient that is about a factor of two larger than that given in Ref. [1].

In Figure 1 (bottom) we plot the field reversal parameter F as a function of time throughout the entire run. As noted, it first settles into a quasi-steady state with $F \sim -0.17$. When the axial driving force is applied and the plasma reaches a flowing quasi-steady state, the average value of F is decreased to ~ -0.27 while Θ remains fixed. During this period the frequency of the oscillations also increases. During and after the decay phase F returns to a value of ~ -0.16, but the frequency of the quasi-periodic oscillations remains somewhat increased in comparison with the evolution before the application of the pulse. Apparently the axial flow leads to change in the dynamo activity that continues after the pulse is turned off. This is inconsistent with Ref. [1], which states that dynamo activity is unaffected by the pulse.

Two measures of dynamo activity are shown in Figure 2. The top figure shows the energy in the radial magnetic field $W_r \sim B_r^2$, which is a measure of the fluctuations that drive the viscosity. During the pulse the amplitude of the fluctuations *decreases* slightly, and the frequency increases. After the pulse the amplitude returns to its original level, but the frequency remains somewhat increased. This seems inconsistent with an increase in dynamo activity resulting from the application of the pulse. The bottom figure plots N_s, a measure of the number of $m = 1$ modes participating in the dynamo, as a function of time. Before and after the pulse $N_s \sim 6 - 7$, indicating a multi-helicity state. During the pulse both the frequency of oscillation and the maximum value of N_s increase. Perhaps the participation of more modes during the pulse results in a stronger dynamo and increased field reversal, but the reason is presently unknown.

We remark that there is one adjustable constant, α_ν, in Equation (4). This has not been tuned to fit the data. Rather, we chose $\alpha_\nu = 15.7$ *a priori* to be the same as used in the estimates of Ref. [1]. Nonetheless, α_ν is related to the correlation length of the fluctuations and is unknown. There is a likely uncertainty in this coefficient is *at least* a factor of two, and therefore correspondingly in the viscosity.

Finally, we have speculated that, since DEBS computes self-consistently the evolution of the fluctuations and their interaction with the plasma, perhaps this by itself can generate an anomalous viscosity that arises from the fluctuations, relying only on self-generated and self-consistent viscosity for flow damping. We have tested this hypotheses by computing the decay phase with $\alpha_\nu = 0$. The code quickly fails due to inadequate spatial resolution caused by insufficient viscous damping. Apparently the physics modeled in DEBS cannot generate sufficient turbulent viscosity to damp the flow as measured in the experiment.

References

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