Spheromak Transients and Energy Confinement

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Outline

• Introduction
  – Spheromak and simulation background
  – NIMROD code

• SSPX modeling
  – Parameters of the computations
  – Comparison with SSPX results
  – Four stages of simulated evolution

• Discussion and Conclusions
The most successful spheromak formation scheme uses electrodes impregnated with vacuum poloidal flux.

- Slow formation was a major conceptual breakthrough [Jarboe, et al., PRL 51, 39 (1983)].
- Most theoretical descriptions have been based on relaxation arguments [Taylor, PRL 33, 1139 (1974); Jarboe, PPCF 36, 945 (1994)].
  - No information on fluctuations
  - Sustainment described by global helicity balance and cascades
- During drive, $T_e < 50$ eV, but during decay or partial drive, $T_e >> 100$ eV has been recorded.

Schematic of the SSPX spheromak experiment at LLNL, with contours of reconstructed symmetric poloidal flux.
While relaxation theory provides insight, numerical computation is required to solve the time-dependent nonlinear equations that describe macroscopic evolution.


- NIMROD simulations of generic spheromaks at 0-\(\beta\) addressed MHD activity underlying formation and sustainment [PRL 85, 4538 (2000), Phys. Plasmas 8, 475 (2001)].
  - Flux amplification results from \(n=1\) MHD activity.
  - Average parallel current is flattened by a dynamo effect.
  - Chaotic scattering of field-lines occurs during sustainment.
  - Closed flux surfaces form during decay.
  - Confinement is not addressed directly.
Recent computations evolve temperature and number density, in addition to the basic resistive MHD model, to investigate energy confinement properties.

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \mathbf{J}) \quad \text{Faraday’s/Ohm’s laws}
\]

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B} \quad \text{low-ω Ampere’s law}
\]

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \rho \mathbf{V} \mathbf{V} \quad \text{flow evolution}
\]

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot \mathbf{D} \nabla n \quad \text{particle continuity}
\]

\[
\frac{n}{\gamma - 1} \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -\frac{p}{2} \nabla \cdot \mathbf{V} + \nabla \cdot n \left[ \chi_{\parallel} \hat{\mathbf{b}} \hat{\mathbf{b}} + \chi_{\perp} (\mathbf{I} - \hat{\mathbf{b}} \hat{\mathbf{b}}) \right] + \eta J^2 \quad \text{(single) temperature evolution}
\]

\[
\hat{\mathbf{b}} \equiv \frac{\mathbf{B}}{|\mathbf{B}|} \quad \text{local magnetic direction vector}
\]

- Braginskii transport coefficients are used for \( \chi_{\parallel} \) (electron), \( \chi_{\perp} \) (ion), and \( \eta \).
- Heating is Ohmic.
Flexibility and accuracy make the NIMROD code (nimrodteam.org) suitable for many configurations.

- Spatial representation is three-dimensional: 2D finite element + 1D Fourier.

- Physics-model flexibility is increasing with continuing code development.

**MST RFP—Reynolds poster**

**LDX—Kesner poster**

**Sheared Slab**

**Two-fluid** (UW; Barnes, CU; Schnack, SAIC)

**Parallel kinetics** (Held, USU)

**Fast ion kinetics** (Kim, UW; Parker, CU)

**Collisional closures** (UW)

**Resistive MHD**
NIMROD’s high-order finite elements make it possible to simulate nonlinear macroscopic evolution with extreme anisotropy and stiffness. [JCP 195, 355 (2004).]

Critical Island Width vs. $\chi_{||}/\chi_{\text{perp}}$

- $W_c$ shows where diffusion time-scales match [Fitzpatrick, PoP 2, 825 (1995)].
- Numerical results measure width needed to affect $T$-profile.

- 5th-order accurate biquartic finite elements were used for this test.
- Resolution requirements are case-dependent.
Simulation of SSPX 4620-4644 Shot Series

**INPUT** (Collisional coefficients are based on Hydrogen and Z=1):

- \( n = 5 \times 10^{19} \, m^{-3} \)
- \( \eta(T) = 411 \left( \frac{1 \, eV}{T} \right)^{3/2} \, m^2/s \)
- \( \chi_\parallel(T) = 258 \left( \frac{T}{1 \, eV} \right)^{5/2} \, m^2/s \)
- \( \chi_\perp = 0.33 \left( \frac{1 \, eV}{T} \right)^{1/2} \left( \frac{1 \, T}{B} \right)^2 \, m^2/s \)
- \( T_{wall} = 0.1 \, eV \)
- \( I_{inj}(t), \psi_{vacuum} \) specified
- \( v = D = 2000 \, m^2/s \)

**OUTPUT:** Everything else

Initial (vacuum) poloidal flux distribution and the NIMROD Mesh of bicubic finite elements representing SSPX (upside down). [Toroidal resolution is \( 0 \leq n \leq 2 \).]
Validity of Collisional Transport

• 0-β MHD results suggest that open-field transport governs confinement during driven conditions.
  – Although individual magnetic field lines do not fill the volume ergodically, we expect Rechester-Rosenbluth collisional transport if the effective mean-free-path is sufficiently small.
  – We can confirm \textit{a posteriori} that collisionless conditions only exist when and where closed flux surfaces form.

• At $n=5 \times 10^{19} \text{ m}^{-3}$ and $T=1 \text{ eV}$,
  \[ v_{Te} \cong 6 \times 10^5 \text{ m/s} \]
  \[ \tau_e \cong 7 \times 10^{-10} \text{ s} \]
  \[ \lambda_e \cong 4 \times 10^{-4} \text{ m} < < L \]

• Scaling $\lambda_e$ with $T^2$ indicates that $\lambda_e$ reaches macroscopic scales between 30 and 50 eV.

• From this we infer that anisotropic thermal conduction is a good model for sustained (open field) conditions and for the transition to closed flux during decay.
The simulated injector current is programmed to approximate the series of SSPX discharges reported in [McLean, *et al.*, PRL 88, 125004-1 (2002)].

A strongly driven phase is followed by decay and then a second, partial drive. [SSPX Data courtesy of H. S. McLean.]

In the early driven phase, the applied potential reaches a few kV. During partial drive, the potential is \(~100\) V.

- There are four stages in the evolution: **1-pinch, 2-driven, 3-decay, 4-partial drive**.
- The peak instantaneous power input reaches \(~1\) GW in the driven stage.

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Toroidal current and magnetic energy evolution from the simulations are similar to that found by CORSICA [Hooper, et al., NF 39, 863 (1999)] fits during the partial-drive stage.

\[ I_{tor} \]

\[ E_{magnetic} \]

- During partial drive in the simulation, the injector circuit provides 14 MW of power and the decay of magnetic energy provides an additional 1.4 MW.
Both simulation and experiment show a quiescent phase when partial drive is applied after a brief period of decay.

Relative poloidal magnetic field fluctuations at the outboard mid-plane position.

- Although conditions are not sustained, partial drive forces fluctuations to smaller amplitude and postpones the emergence of the $n=2$ mode.

- Another simulation series that includes all $n \leq 5$ produces fluctuations at larger $n$-values that reduce $T$ by $\sim 25\%$ late in time.

Observed and computed temperatures are highest during the quiescent phase.
Four Stages of Simulated Evolution: 1-Symmetric Pinch ($t<0.08$ ms)

- Injected toroidal flux pushes plasma and poloidal flux into the flux-conserver region.

With temperature-dependent resistivity, current is carried by a thin plasma layer until the symmetric distribution becomes MHD unstable.

The vacuum poloidal flux is stretched and compressed, but there is no flux amplification at this stage. The Poincaré plot shows some dynamically formed closed-flux surfaces.
Four Stages of Simulated Evolution: 2-Driven ($0.08 \leq t \leq 0.12$ ms)

- Similar to $0-\beta$ simulation results, saturation of the $n=1$ mode leads to redistribution of parallel current through the MHD dynamo effect.

The symmetric component of parallel current density is positive throughout most of the flux conserver region.

- Regions of positive power density transfer energy from the symmetric magnetic field to fluctuations; negative power density drives symmetric current.

- Dynamo activity was measured in SPHEX, but $n>1$ was thought to represent separate activity [al-Karkhy, et al., PRL 70, 1814 (1993)].
Four Stages of Simulated Evolution: 2-Driven (0.08 ≤ t ≤ 0.12 ms)

- Also similar to 0–β simulation results, saturation of the $n=1$ mode (including reconnection) converts toroidal magnetic flux into poloidal magnetic flux.

The symmetric component of poloidal magnetic flux has a new minimum value, indicating ~200% flux amplification. However, magnetic field-lines show chaotic scattering.

With chaotic magnetic field-lines, parallel conduction transports heat to the walls, and the temperature is essentially uniform with a maximum value of ~35 eV.
Further Comments on the Driven Stage

• Heat flux diagnostics show that convection magnitude is large and may affect confinement.

• The evolving number density is now being used in the flow velocity and temperature equations.
  – $D$ is large ( $\tau_n \leq \tau_E$-observed).
  – Driven stage is quite violent.
  – Particle transport is one of the least understood aspects of SSPX, according to experimentalists.

Contours of number density at $\phi=0$ show large variations during peak current, even with the large value of $D$. 

\[ n \ (m^{-3}) \]

- $5.0E+20$
- $4.3E+20$
- $3.7E+20$
- $3.0E+20$
- $2.3E+20$
- $1.7E+20$
- $1.0E+20$
- $3.6E+19$
Four Stages of Simulated Evolution: 3-Decay ($0.12 \leq t \leq 0.5$ ms)

- With the drive off, temperature on the outer field-lines decreases rapidly, enhancing resistive reconnection and the formation of flux surfaces.

Energy confinement improves while Ohmic heating continues through magnetic energy decay. At 0.5 ms, the temperature at the magnetic axis reaches 76 eV.
Thermal transport changes character from driven to decaying conditions (0.12 ms top row and 0.5 ms bottom row).

conductive $<q_{\parallel} B_{pol}>$

conductive $<q_{\perp pol}>$

convective $<2nTV_{pol}>$

Note the different exponential scales for the magnitudes of the three heat vectors.
Four Stages of Simulated Evolution: 4-Partial Drive (0.5 ms≤t)

- With partial drive, the flux surfaces persist longer, and the peak temperature is higher.

Topologically, the partial drive stage is similar to decay.

The ‘shot’-maximum temperature of 111 eV occurs at \( t = 1.08 \) ms.

- The second drive delays the onset of resonant \( n=2 \) activity, and it heats plasma surrounding the flux surfaces.
Four Stages of Simulated Evolution: 2-Driven vs. 4-Partial

- Although the partial-drive stage has half of the injected current as the driven stage, the dynamo power density is two orders of magnitude smaller. [Note the contour-level scales in the following plots.]

- The ratio of electrical conductivities at the respective temperatures is less than 10.
Safety factor and parallel current profiles suggest that the improvement from partial drive results from avoidance of low-order rational surfaces (~Woodruff and McLean, recent).

Safety factor profiles from the decay and partial drive computations shown at t=1.01 ms.

Parallel current density profiles shown at t=1.01 ms.
Discussion

• An interplay of inductive effects and temperature-dependent transport coefficients produces the low-fluctuation, high-confinement states.
  – The \( n=1 \) mode of the open-field current channel decays rapidly when the drive is removed—the open-field plasma cools, and the pinch current subsides.
  – Low-resistivity plasma within the hot flux surfaces retains toroidal current associated with the \( n=1 \)-generated poloidal flux. [At 60 eV, for example, the symmetric current would require about 8 ms to decay resistively.]
  – The influence of the MHD activity on the magnetic topology during drive and decay are consistent with earlier \( 0-\beta \) simulation results (Finn, et al., PRL 85, 4538, 2002 and Sovinec, et al., PoP 8, 475, 2001).

• The realistic parameters and collisional temperature dependencies make the MHD results quantitatively consistent with SSPX results (McLean, et al., PRL 88, 125004, 2002).
  – Temperature evolution
  – Magnetic fluctuations of \( \sim 1 \% \) during partial drive
  – Magnetic energy decay during partial drive

• The resonant fluctuations during partial should be analyzed with respect to their impact on confinement and not as a mechanism for current drive.
Conclusions

• Evolving the complete system with temperature-dependent transport coefficients allows us to assess confinement quality during different stages of spheromak operation.

• Transients play a crucial role; thus, modeling injector current programming allows detailed comparison of theory and experiment.

• Relating the $0-\beta$ simulation results to the recent results and from the comparison of the experimental observations with the simulations, we see that only the initial phase of the standard two-stage SSPX operation has the characteristics of full sustainment.

• The sustaining $n=1$ MHD mode is not excited during the quiescent phase. The success of partial drive provides optimism for tailoring pulsed spheromak operation.
Acknowledgments

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