Toroidal Geometry Effects in the Low Aspect Ratio Ratio RFP

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Objective

Determine the influence of toroidal geometry on low-$\beta$ reversed-field pinch configurations.

Outline

I. Introduction
   A. Background
   B. Geometric considerations
   C. Modeling

II. Aspect ratio scans
   A. $F-\Theta$
   B. Magnetic fluctuations
   C. Spectrum width

III. Laminar RFP states
   A. Single helicity conditions
   B. Quasi-single helicity conditions

IV. Remaining questions

V. Conclusions
Background

- Most numerical simulations and analytic computations for the RFP have been performed in periodic linear geometry.

- This is usually a good approximation.

  - Since $q<1$, pressure gradients cannot stabilize tearing modes (assuming $p$ decreases with $r$). [Glasser, Greene, and Johnson, Phys. Fluids 18, 875 (1975).]

  - Strong nonlinear coupling among resonant fluctuations of different poloidal index $m$ is a characteristic of standard RFP operation. [Ho and Craddock, Phys. Fluids B 3, 721 (1991).]

- A laminar version of the RFP dynamo, known as the "single-helicity state," exists at sufficient dissipation levels in periodic linear geometry. [Finn, Nebel, and Bathke, Phys. Fluids B 4, 1262 (1992); Cappello and Escande, PRL 85, 3838 (2000).]

  - Usual nonlinear coupling among different helicities is absent.

  - Toroidal geometry effects can make a qualitative difference in these conditions, due to linear coupling of different $m$. 
Geometric Considerations

- Many low-order helicities are resonant in a typical RFP $q$-profile.

- Close spacing of the rational surfaces and the global nature of the dominant tearing modes allow for strong nonlinear coupling in standard operation.
• Well known for tokamaks, toroidal geometry leads to **linear** coupling among helicities of different \( m \).

• The gradient operator contains

\[
\frac{1}{R} \frac{\partial}{\partial \varphi} = \frac{1}{(R_0 + r \cos(\theta))} \frac{\partial}{\partial \varphi}
\]

\[
= \frac{1}{R_0} \left( 1 - \varepsilon \frac{r}{a} \cos(\theta) + \left( \varepsilon \frac{r}{a} \right)^2 \cos^2(\theta) - \ldots \right) \frac{\partial}{\partial \varphi}
\]

where \( \varepsilon = a/R_0 \) and the \( \cos(\theta) \) terms lead to the coupling.

• The Shafranov shift introduces poloidal asymmetries in the equilibrium, also leading to linear coupling.

• For an RFP \( q \)-profile we can expect the strongest poloidal coupling to occur between \( m=0 \) and \( m=1 \) helicities.
Modeling

To investigate the electromagnetic activity, we solve the zero-$\beta$ resistive MHD equations in circular cross-section, toroidal and periodic linear geometries using the NIMROD simulation code, http://nimrodteam.org.

$$\frac{\partial V}{\partial t} + V \cdot \nabla V = \frac{1}{\rho} J \times B + \nabla \cdot (\nu \nabla V)$$

$$E = -V \times B + \eta J$$

$$\frac{\partial B}{\partial t} = -\nabla \times E$$

- Density is uniform, though flow is not incompressible.
- Resistivity and viscosity are essentially uniform. [$S=2500$ and $P_m \equiv \mu_0 \nu / \eta = 1 - 100$]
- Voltage is adjusted to maintain the desired plasma current. The time-scale for the feedback is comparable to the tearing time to avoid excitation of surface currents.
- Two-fluid effects may be important.
  - The drift ordering is more realistic for RFPs than the MHD ordering even at small $\beta$.
  - Worth further investigation.
  - Two-fluid capabilities in NIMROD are being developed through the PSACI project.
• Numerical parameters:
  • Most of the simulations reported here have $0 \leq n \leq 42$.
  • Some of the simulations for laminar conditions have $0 \leq n \leq 21$.
  • NIMROD uses finite elements to represent the poloidal plane. Simulations for the aspect ratio scan were run with a 48x48 or 64x64 (radial x azimuthal) mesh of bilinear finite elements.
  • Where viscosity is scanned to suppress nonlinear activity, a 16x24 or 16x32 mesh of bicubic elements is used for a better representation of the magnetic field. [See "Nonlinear Fusion Magneto-Hydrodynamics with Finite Elements," Sherwood 2000, in http://nimrodteam.org/presentations.]
Aspect Ratio Scans in Toroidal and Periodic Linear Geometries

- Results on field reversal from dynamo action are similar in the two geometries, even at very low aspect ratio.

Comparison of time-averaged reversal parameter ($F$) resulting from simulations in (a) toroidal geometry and (b) periodic linear geometry at $S$=2500 and $P_m$=1.
• Magnetic fluctuation levels are also comparable.

<table>
<thead>
<tr>
<th>geometry</th>
<th>$R/a$</th>
<th>$\Theta$</th>
<th>$\frac{n &gt; 0 \text{energy}}{\text{total energy}}$</th>
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<tr>
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<td>1.1</td>
<td>1.6</td>
<td>0.091</td>
</tr>
<tr>
<td>linear</td>
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<td>1.6</td>
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<td>1.6</td>
<td>0.084</td>
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<tr>
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</tr>
<tr>
<td>linear</td>
<td>2</td>
<td>1.8</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Results are averaged over 1-2 tenths of a global diffusion time.
Magnetic energy spectra plotted vs. \( n \) and summed over \( m \) for the two geometries are often nearly indistinguishable for standard multi-helicity states.

Magnetic fluctuation energy spectra for a) toroidal geometry and b) periodic linear geometry showing the temporal average (red) and \( \pm \) one standard deviation (blue) for \( R/a=1.75, P_m=1, \Theta=1.8 \).
• The spreading of the magnetic spectrum with $R/a$ reported by Ho, et al. ["Effect of aspect ratio on magnetic field fluctuations in the reversed-field pinch," Phys. Plasmas 2, 3407 (1995).], is also observed in toroidal geometry.

• Each $W_n$ is summed over $m$.

• Nonlinear interaction seems to be more easily suppressed in toroidal geometry.

Simulation results on $N_s \equiv \left( \sum_n W_n \right)^2 / \sum_n W_n^2$ for $\Theta=1.6$, $P_m=1$ simulations. At $R/a=1.5$, $q(0)$ is slightly greater than $1/3$. 
Laminar RFP States

- As viscosity is increased there is a transition to steady or near-steady states.

- Cappello and Escande have established that this transition is more dependent on the Hartmann number \( H \equiv S P_m^{-1/2} \) than the Lundquist number.

Transition to laminar states in periodic linear geometry with \( R/a=4 \). [Cappello and Escande, "Bifurcation in Viscoresistive MHD: The Hartmann Number and the Reversed Field Pinch," PRL 85, 3838 (2000).]
• A transition to laminar behavior also occurs in toroidal geometry as $P_m$ is increased. The following figure shows the transition in the toroidal $R/a=1.75$, $\Theta=1.8$ case after $P_m$ is increased from 1 to 10.

• Plotted vs. $n$ and summing over $m$, the spectrum has the appearance of a single helicity state ($E_{(m,n)}=0$ if $m/n\neq 1/n_p$, where $n_p$ is the toroidal index at the peak of the spectrum).
- Poincaré surfaces of section for $\mathbf{B}$ show that the final state in toroidal geometry is not single-helicity, however.

Results from a) toroidal geometry and b) periodic linear geometry with $P_m=10$, $R/a=1.75$, $\Theta=1.8$. 
The magnetic spectrum for the toroidal case shows that while the $m/n=1/n_p$ helicities have a large fraction of the energy, linear poloidal coupling (among the same $n$-values) is also quite significant.
• When $P_m$ is increased another order of magnitude, the toroidal simulation loses reversal. With $m=0$ fluctuations no longer resonant, a helical island chain forms in the interior.
In a periodic linear geometry simulation with $P_m=10$, $\Theta=1.65$, and $R/a=1.75$, the final state is steady, but not single helicity.
• These "quasi-single-helicity" states may show a coherent island structure by having a sufficiently large perturbation [Escande, et al., "Chaos Healing by Separatrix Disappearance and Quasisingle Helicity States of the Reversed Field Pinch," PRL 85, 3169 (2000)].
The spectrum in toroidal geometry with $P_m=10$, $\Theta=1.6$, and $R/a=1.75$ is similar, and a helical structure may be evident. However, $F>0$ and the lack of $m=0$ resonance is important.
Remaining Questions

1. What value of $R/a$ is sufficiently large for the formation of laminar states with large island structures?

2. Can the strong poloidal coupling at low $R/a$ be used to suppress fluctuations through sheared flow?
Conclusions

1. In typical multi-helicity RFP operation, toroidal geometry plays a minor role in comparison to nonlinear coupling.

2. In laminar conditions, toroidal geometry can make a qualitative difference. Conditions producing nonstochastic magnetic field in periodic linear geometry may have large regions of stochastic field in toroidal geometry.

3. The narrowing of the fluctuation spectrum as $R/a$ is decreased does not indicate a transition to single helicity when toroidal geometry effects are considered.

4. The strong coupling at low $R/a$ may be an opportunity for reducing fluctuation levels through shear flow. This remains to be explored.

This poster will be available through our web site, http://nimrodteam.org, shortly after the meeting.