A Comparison of MHD Activity and Nonlinear Dynamics in RFPs and Electrostatically Sustained Spheromaks

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MAJOR FINDINGS

• In the past, dynamo in RFPs and relaxation in spheromaks have been compared loosely.
• With the correct geometric perspective, MHD simulations show that the analogy holds extremely well for specific characteristics:
  • Free energy source
  • Linear instability
  • Feedback on symmetric fields
• An unrealistically small $R/a$ RFP simulation illustrates where quantitative differences arise.
  • Spectra
  • Fluctuation levels
• The analogy does not extend to magnetic topologies, however, due to the electrode-penetrating flux in the spheromak.
The comparisons are made with numerical solutions of the zero-\(\beta\) resistive MHD equations, which are appropriate for studying current driven activity in both devices.

\[
\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = \frac{1}{\rho} \mathbf{J} \times \mathbf{B} + \nabla \cdot (\nu \nabla \mathbf{V})
\]

\[
\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]

- Walls are ideal, except for allowing the necessary sources of magnetic flux to drive the discharges.
- Only considering sustained conditions.
- \(S\) is \(10^3\)-\(10^4\), except where noted.
- Spheromak and new RFP simulations have been performed with the **NIMROD** code, [http://nimrodteam.org](http://nimrodteam.org).
Flux-core spheromaks and gun-driven spheromaks have the same topology, but the RFP comparison is more easily visualized with the flux-core configuration.
When comparing RFPs and spheromaks, the direction of the applied electric field has primary importance.

- Spheromak $z$-direction :: RFP $\phi$-direction
Current density is strongest in the direction of the applied electric field.

- Pinching is perpendicular to this direction.
- This establishes the current gradient and source of free energy for breaking azimuthal symmetry.

**Line-tied Pinch**

**R/a=2 Periodic Pinch**

\[ \lambda = \frac{\mu_0 a J_\parallel}{B} \]
In both configurations, the free energy most easily excites an azimuthal mode number 1 instability.

- "m" number 1 for RFPs
- "n" number 1 for spheromaks

**Line-tied "n"=1 mode**

**Periodic (1,4) mode**
• It is well known that saturation of the $m=1$ modes brings about **field reversal** in RFPs.

• Saturation of the analogous current driven instability leads to **flux amplification** in spheromaks.

**n=0 Poloidal Flux Contours for a Spheromak**

Unstable Pinch

Saturated State

Spheromak flux amplification is a more extreme manifestation of the same saturation mechanism.
A spheromak-like simulation with periodic ends bridges the gap between the spheromak and the RFP.

- $R/a=1/2\pi$
- $\Theta=\mu_0aI/2\Phi=6.5$

The resulting flux amplification is 170% for both.
- Line tying does little in the final state.

- From RFP studies, we know that the spectrum narrows as $R/a$ is decreased.$^1$
- Larger normalized current drives larger fluctuations in spheromaks (~10% vs ~1% in RFPs).

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As $S$ is increased in RFP simulations, the MHD activity becomes progressively more intermittent.\footref{footnote:1}

As $S$ is increased in RFP simulations, the MHD activity becomes progressively more intermittent.\textsuperscript{2}

In spheromak simulations, the final state is steady at low values of $S (=1000)$. When $S$ is increased (5000), the sustained states are nonsteady, often showing limit cycle behavior.
Topology of Magnetic Field Lines

With a conducting wall, and in the absence of field errors, a single magnetic trajectory ergodically samples the entire volume of a standard RFP.

- The trajectory is *stochastic*.

530 intercepts of a constant-$\phi$ plane over a single integration path of approximately 25,000 $R$.

- Toroidal geometry RFP simulation, $R/a=2$.
- $\Theta = 1.8$, $F = -0.1$. 
Open field lines in spheromaks may be given unique labels.

- Trajectories are described by *chaotic scattering*. 

![Graphs showing field lines at different Z levels](image-url)
Because individual lines may be identified in a spheromak, we can find the distribution of lengths.

- An exponential distribution, observed in typical cases, is characteristic of hyperbolic scattering.
Plotting field-line length as a function of the starting position for each trajectory shows the fractal mixing of lines of different length.

- Thus, there is no region of exclusively long lines.
RFP configurations achieve closed flux surfaces in single helicity and quasi-single helicity states. Spheromak simulations show a similar state with large closed flux surfaces just above the pinch stability threshold.

- Flux surfaces and central current have helical distortions.
- Entrained poloidal field is very weak, and the safety factor at the magnetic axis is of order 10.

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In steady state, the current distribution and magnetic topology are related through the parallel component of Ohm's law.

When the only parallel electric field is $\eta J$, the difference in electrostatic potential between two points along a field line is

$$\delta \chi (L) = -\int_0^L E \cdot B \frac{dL'}{B} = -\int_0^L \frac{\eta}{\mu_0} B dL'$$

where the path of integration follows the field line trajectory.

**On closed flux surfaces,**

$$\langle f \rangle_\Psi = \frac{\oint f dL/B}{\oint dL/B}$$

$$\langle \eta \lambda B^2 \rangle_\Psi = -\frac{\oint dL\hat{\mathbf{b}} \cdot \nabla \chi}{\oint dL/B} = 0$$

**Therefore:**

- Within a surface, $\lambda$ changes sign, *or*
- In the limit of $\mathbf{J} \times \mathbf{B} = \mathbf{0}$, $\lambda=0$.
- Flux surfaces are analogous to stellarator flux surfaces.
For open field topologies,

Integrating from one electrode to the other with the approximately uniform resistivity,

\[ \delta \chi(L) = V \approx -\frac{\eta}{\mu_0} \langle \lambda B \rangle_B L \]

where \( \langle f \rangle_B \equiv \frac{1}{L} \int_0^L f dL' \)

Therefore:

- The ensemble average of \( \langle \lambda B \rangle_B \) is proportional to the ensemble average of \( L^{-1} \equiv 1/L^* \).

Also,

- Average parallel current tends to approach the first eigenvalue of \( \nabla \times \mathbf{B} = \mu \mathbf{B} \) with homogeneous boundary conditions as the applied voltage becomes large.\(^5\)

As voltage is increased and the toroidally averaged $B$ approaches the first eigenfunction, the average field-line length increases through spreading of the distribution.
CONCLUSIONS

• The direction of the applied electric field is the most important consideration for making the RFP-spheromak analogy.

• Though spheromak sustainment has previously received less investigation than RFP dynamo, we can draw from the MHD studies of RFPs.
  • The dominant $n=1$ spheromak perturbation is like the core-resonant RFP modes.
  • The small effective aspect ratio narrows the spheromak spectrum.
  • Flux amplification is a more dramatic realization of field reversal.
  • Larger magnetic fluctuations result from larger normalized currents.

• Open field lines change the nature of the magnetic topology.
  • Fractal mixing means that long field lines are always close to short field lines.
  • RFPs with field errors likely have short parallel connections from the core to the wall, also.