Ohmic Current Drive in NIMROD Simulations

Carl Sovinec, *University of Wisconsin-Madison*

October 28, 2005

One basic aspect of simulating magnetic confinement configurations with MHD and extended-MHD codes is that current drive must be represented in some way. In NIMROD, we have the ability to separate a symmetric steady-state component from the dynamic solution, and some fraction of the current drive may be represented in the steady-state component. The intent of this note is to provide clarification and one perspective on the NIMROD formulation.

In NIMROD, the combined Faraday's/Ohm's law equation is similar to other equations in that after expanding fields into steady-state and evolving parts, $\mathbf{B}(t) \Rightarrow \mathbf{B}_{ss} + \tilde{\mathbf{B}}(t)$ for example, terms that contain only steady-state factors are cancelled. Here, the implication is that the steady-state electric field is curl-free (or nearly curl-free, see below). The evolving part of the magnetic field then obeys

$$
\begin{aligned}
\frac{\partial \tilde{\mathbf{B}}}{\partial t} &= -\nabla \times \tilde{\mathbf{E}} \\
&= -\nabla \times \left[\tilde{\eta} \mathbf{J}_{ss} + \eta_{ss} \tilde{\mathbf{J}} + \tilde{\eta} \tilde{\mathbf{J}} - \tilde{\mathbf{V}} \times \mathbf{B}_{ss} - \mathbf{V}_{ss} \times \tilde{\mathbf{B}} - \tilde{\mathbf{V}} \times \tilde{\mathbf{B}} \right]
\end{aligned}
\tag{1}
$$

for the choice of resistive MHD Ohm's law. One can work backwards and substitute the difference between total fields and the respective steady-state part to find the following evolution equation for the total field:

$$
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\eta \mathbf{J} - \mathbf{V} \times \mathbf{B} - \eta_{ss} \mathbf{J}_{ss} + \mathbf{V}_{ss} \times \mathbf{B}_{ss} \right]
$$

or

$$
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\eta \mathbf{J} - \mathbf{V} \times \mathbf{B} - \mathbf{E}_{ss} \right] \tag{2}
$$

where $\mathbf{E}_{ss} = \eta_{ss} \mathbf{J}_{ss} - \mathbf{V}_{ss} \times \mathbf{B}_{ss}$.

If the steady component of **E** is curl-free, the evolution of **B** from Eq. (2) is what we expect from the resistive MHD equations, i.e. Eq. (2) without the $\mathbf{E}_{ss}$ term. In the absence of fluctuations, average parallel current requires the application of an electric field (assuming nonzero resistivity), and for toroidal current, there must be net loop voltage. Loop voltage can be applied to the evolving component by specifying a tangential $\tilde{\mathbf{E}}_{wall}$ in the surface integral of the weak form of Faraday's law,

$$
\int_R \mathbf{a} \cdot \frac{\partial \tilde{\mathbf{B}}}{\partial t}\, dVol = -\int_R \nabla \times \mathbf{a} \cdot \tilde{\mathbf{E}}\, dVol + \int_{\partial R} \mathbf{a} \cdot \tilde{\mathbf{E}}_{wall} \times d\mathbf{S} \ , \tag{3}
$$

for all functions **a** in the set of basis functions. The surface integral is a physically controllable quantity; it is the rate at which magnetic flux enters or leaves the domain. Thus, we are free to specify the tangential component of $\tilde{\mathbf{E}}_{wall}$, unlike $\tilde{\mathbf{E}}$ in the interior of the domain or the normal component of $\tilde{\mathbf{E}}$ at the wall. Mathematically, the surface integral is a natural condition on the tangential components of $\tilde{\mathbf{B}}$ unless they are directly specified otherwise. The "loop_volt" and "e_vertical" input parameters for NIMROD control the applied tangential components of $\tilde{\mathbf{E}}_{wall}$. When a tangential $\tilde{\mathbf{E}}_{wall}$ is applied, the evolution of the current density profile and the amount of net resulting current are determined by the dynamics of the system, including the spatial average of $\tilde{\eta}\tilde{\mathbf{J}} - \tilde{\mathbf{V}} \times \tilde{\mathbf{B}}$.

If a NIMROD computation has a self-consistent ($\nabla \times \mathbf{E}_{ss} = \mathbf{0}$) steady state with nonzero current density, the total current density is the sum of the steady component $\mathbf{J}_{ss}$ and any $\tilde{\mathbf{J}}$ (including net toroidally symmetric current) generated by the dynamics. When the configuration is stable and "loop_volt" and "e_tangential" are zero, the net current remains the same as the current in the steady state. If "loop_volt" and "e_tangential" are specified to be nonzero, net current will likely be generated in the profile of $\tilde{\mathbf{J}}$ which increases or decreases the total current. However, it's generally not the sum of the steady current and the current generated by the same values of "loop_volt" and "e_tangential" without the steady background, because the system is nonlinear. Another instructive Gedanken experiment is to consider a very unstable case with temperature-dependent resistivity and anisotropic energy transport without applied $\tilde{\mathbf{E}}_{wall}$. The loop voltage measured at the wall remains the value associated with $\mathbf{E}_{ss}$, but a sudden loss of confinement leads to increasing resistivity. There will be a positive increase of the net E_{tor} in the interior, and the resulting curl of **E** (total) will reduce the poloidal field and net toroidal current, despite the fact that the loop voltage applied at the surface (from the curl-free $\mathbf{E}_{ss}$) is fixed. The induced $\tilde{\mathbf{J}}$ then cancels part of $\mathbf{J}_{ss}$.

Specifying net current instead of net voltage requires a modification of the boundary conditions on the evolution of $\tilde{\mathbf{B}}$, regardless of the steady state. We would have to ensure that the boundary conditions on $\tilde{\mathbf{B}}$ are such that

$$\oint \left(\mathbf{B}_{ss} + \tilde{\mathbf{B}}_{wall}\right) \cdot d\mathbf{l}$$

matches a specified value (or waveform) along all appropriate paths over the surface of the domain. This can be done in principle by changing the condition on some component (vector direction and Fourier) of $\tilde{\mathbf{B}}_{wall}$ from a natural condition to an essential condition. This is straightforward for coaxial current injection (as in SSPX), where the path runs in the toroidal direction and the toroidally symmetric, toroidal component of $\tilde{\mathbf{B}}_{wall}$ is only specified over the portion of the wall representing the gap between the electrodes. In the simply connected geometry, this forces the specified amount of current to enter the domain from the electrode surrounded by the gap. Precisely specifying the net toroidal current in a toroidal configuration is much more difficult, because the integral is a nonlocal condition—only a weighted sum of one of the tangential components of $\tilde{\mathbf{B}}_{wall}$ is determined. It is far easier in a code with a spectral representation of the poloidal direction where the covariant components of $\tilde{\mathbf{B}}$ are expanded, so that poloidal variations in the metric drop-out of the path integral, and we're just left with a condition on the *m*=0 component:

$$\begin{aligned}\mu_0 I_\phi(t) - \oint(\mathbf{B}_{ss})\cdot\frac{\partial\mathbf{R}}{\partial\theta}d\theta &= \oint\left(\tilde{\mathbf{B}}_{wall}\right)\cdot\frac{\partial\mathbf{R}}{\partial\theta}d\theta \\ &= \oint\left(\tilde{B}_{wall_\theta}\nabla\theta\right)\cdot\frac{\partial\mathbf{R}}{\partial\theta}d\theta \\ &= \oint\tilde{B}_{wall_\theta}d\theta\end{aligned}$$

With a poloidal mesh, it is easier to use the natural conditions and specify the toroidal component of $\tilde{\mathbf{E}}_{wall}$ according to some feedback scheme on the net toroidal current. This is the purpose of the NIMROD input parameters "i_desired," "loop_rate," and "loop_rate2," which are used in

$$\frac{V^n - V^{n-1}}{\Delta t} = \text{loop\_rate}\left(\text{i\_desired} - I_\phi^{\,n-1}\right) - \text{loop\_rate2}\left(\frac{I_\phi^{\,n-1} - I_\phi^{\,n-2}}{\Delta t}\right)$$

as borrowed from Bill Ho's version of DEBS, except that in NIMROD, $V \equiv \oint\tilde{E}_{wall_\phi}Rd\phi$. The largest difficulty with this approach is finding appropriate parameters for a given situation. (Note that the two rate parameters have different units.) Provided that the feedback is slow relative to MHD propagation, the temporal differencing doesn't cause added difficulty.

Some consideration of the situation where $\nabla\times\mathbf{E}_{ss}\neq\mathbf{0}$ is also appropriate, since it frequently occurs in our nonlinear tokamak computations. Here, we often use resistivity values and profiles that are not accurate for an experiment, and the experiment may have applied or natural sources of current drive that are outside the scope of the specified Ohm's law. For resistive MHD, Eq. (2) implies that the total **B** will continue to evolve until $\mathbf{E} = \eta\mathbf{J} - \mathbf{V}\times\mathbf{B} - \mathbf{E}_{ss}$ is free of curl. Interpreted as an electron equation of motion, $d\mathbf{V}_e/dt \sim -\mathbf{E} - \mathbf{E}_{ss} - \mathbf{V}\times\mathbf{B} + \eta\mathbf{J}$ with negligible electron inertia, the modified Ohm's law implies the existence of a new force that is proportional to $-\mathbf{E}_{ss}$ on the electrons. It may drive current, or it may lead to other effects, as determined by the dynamics. Many studies have used this interpretation (including NF **39**, p. 777, 1999), and it is reasonable if the drive remains at fixed location and direction. It may also be suitable when the drive changes if the portion of $\mathbf{E}_{ss}$ with nonzero curl is small, i.e. $|\mathbf{B}|/|\nabla\times\mathbf{E}_{ss}| >> \tau$, where τ represents the time-scale of interest. This may not be the case for net bootstrap drive, and the assumption of fixed-position forcing may be particularly bad for ELM simulations, where the sharp edge pressure profile collapses and the gradient moves inward. Some model of the net bootstrap drive is likely necessary and is part of the transport-level information that we need to consider in our simulations.