Simulated Flux-Rope Evolution during Non-Inductive Startup in Pegasus

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CPTC Seminar October 14, 2013
How do flux-ropes driven by localized injectors in Pegasus produce tokamak-like configurations?

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Introduction: Observations of solar flares and coronal mass ejection events have prompted many studies of magnetic flux ropes.

Double solar prominence eruption on Nov. 16, 2012 imaged by NASA’s Solar Dynamics Observatory.

Flux ropes are broadly defined as columns of axial magnetic field wrapped by azimuthal field.

- Separation of axial and radial spatial scales is implied; ropes have some degree of radial localization.
- General definitions allow zero-net-current columns and current-carrying columns.
  - Azimuthal-$B$ is strictly localized in the former but not in the latter.
  - Experiments usually generate ropes with net current.


Flux ropes are generated in some non-inductive approaches to current drive and startup in tokamaks.

- Tests in CDX and CCT used emissive cathodes in vacuum field with toroidal and poloidal components [D. S. Darrow, et al. PFB 2, 1415 (1990)].
- Experiments on Pegasus also use localized injectors, and two geometrically different configurations have been tested.

Visible-light images from Pegasus in filament, sheet, and diffuse operation. (Eidietis, JFE)

- Pegasus results with the divertor-injector configuration established distinct-filament, sheet, and diffuse tokamak-like discharges [N. W. Eidietis, J. Fusion Energy 26, 43 (2007)].
- The outboard-injector configuration includes PF induction for force-balance and additional current drive [Battaglia, NF 51 073029 (2011)].
Our modeling simplifies the injectors, making them localized volumetric sources of helicity and heat.

- We focus on basic flux-rope dynamics and consider the first Pegasus configuration with a single source and without PF induction.
- In the experiment, DC voltage is applied from the plasma-injector aperture to an anode plate or the tank.
  - Magnetic helicity is injected via the surface contribution \(-2 \oint \chi \mathbf{B} \cdot d\mathbf{S}\).
- Our simulations model the injectors with a localized source \(\mathbf{E}_{\text{inj}} = \eta \mu_0^{-1} \lambda_{\text{inj}} \mathbf{B}\) in Ohm’s law, oriented parallel to the desired current direction, to source helicity \(+2 \int \mathbf{E}_{\text{inj}} \cdot \mathbf{B} d\text{Vol}\).
- Localized ion heat is also applied.

Isosurface of simulated \(\lambda = \mu_0 J_\parallel / B\) at low injection levels with dark shading in the source-density region.
Our computations solve resistive-MHD and low-frequency two-fluid models starting from vacuum field and ‘cold fluid.’

\[
\frac{\partial n}{\partial t} = -\nabla \cdot \left( n \mathbf{V} - D_n \nabla n + D_h \nabla^2 n \right)
\]

\[
\begin{align*}
\frac{\partial}{\partial t} \left( \frac{m n}{2} \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V} \right) &= \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \left( m n \mathbf{W} \right) \\
\mathbf{W} &= \nabla \mathbf{V} + \nabla \mathbf{V}^T - \left( \frac{2}{3} \right) (\nabla \cdot \mathbf{V}) \mathbf{I}
\end{align*}
\]

\[
\frac{2n}{3} \left( \frac{\partial}{\partial t} T_e + \mathbf{V}_e \cdot \nabla T_e \right) = -n T_e \nabla \cdot \mathbf{V}_e + \nabla \cdot \left[ (\kappa_{\parallel e} - \kappa_{\perp e}) \hat{\mathbf{b}} \hat{\mathbf{b}} + \kappa_{\perp e} \mathbf{I} \right] \cdot \nabla T_e + n \sigma (T_i - T_e) + \eta \mathbf{J}^2
\]

\[
\frac{2n}{3} \left( \frac{\partial}{\partial t} T_i + \mathbf{V}_i \cdot \nabla T_i \right) = -n T_i \nabla \cdot \mathbf{V}_i + \nabla \cdot \left[ (\kappa_{\parallel i} - \kappa_{\perp i}) \hat{\mathbf{b}} \hat{\mathbf{b}} + \kappa_{\perp i} \mathbf{I} \right] \cdot \nabla T_i + n \sigma (T_e - T_i) + Q_i
\]

\[
\frac{\partial}{\partial t} \mathbf{B} = -\nabla \times \left[ \eta \left( \mathbf{J} - \frac{\lambda_{inj}}{\mu_0} \mathbf{B} \right) - \mathbf{V} \times \mathbf{B} + \frac{1}{n e} (\mathbf{J} \times \mathbf{B} - \nabla p_e) \right]
\]

\[
\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}
\]

- Neutrals, ionization, and recombination are not modeled.
- The NIMROD code (nimrodteam.org) is used to solve these systems.
Simulation parameters approximate conditions in the first non-inductive startup experiments in Pegasus.

Transport:

- Thermal conduction modeling includes magnetization, which is larger in the current filaments than between them.

\[
\kappa_{\parallel\alpha} = \frac{nT_\alpha \tau_\alpha \gamma_0}{m_\alpha \delta_0} \quad \kappa_{\perp\alpha} = \frac{nT_\alpha \tau_\alpha \gamma_1 x_\alpha^2 + \gamma_0}{m_\alpha \tau_\alpha x_\alpha + \delta_1 x_\alpha^2 + \delta_0} \quad x_\alpha = \Omega_\alpha \tau_\alpha
\]

- The thermal equilibration rate includes \( T \)- and \( n \)-dependence.
- Resistivity evolves with local temperature, \( \eta(T_e) \sim T_e^{-3/2} \), with \( \mu_0^{-1} \eta(1 \text{ eV}) = 100 \text{ m}^2/\text{s} \).

Initial conditions:

- Density of \( n = 1 \times 10^{19} \text{ m}^{-3} \)
- \( I_{TF} = 35 \text{ kA}, B_\nu = 3.7 \text{ mT} \)

- The injection magnitude (\( \lambda_{\text{inj}} \)) is increased linearly in time.
- Temperatures and \( B_\phi \)-values are relaxed along boundaries.
Phases of development: As $I_p$ builds, the activity and path of the rope change qualitatively.

Toroidal plasma current from MHD computation

Relative magnetic fluctuations measured near midplane

Initial current path

Early ring formation

Toroidal winding
Vacuum-field winding that directs the flux rope to have multiple passes leads to island-coalescence behavior.

Sketches from Finn&Kaw, PF 20, 72 (1977) for a periodic system.

Parallel current from the early phase of a Pegasus simulation.

- Pegasus ropes are neither periodic nor sinusoidal.
- Merging of different passes leads to ring formation. [O’Bryan, PoP 19, 080701.]

- Multiple passes are required for relaxation into diffuse discharges [Eidietis, PhD thesis].
Large-scale reversal of $B_z$ near the central column leads to $I_p/I_{inj}$ exceeding vacuum winding in the experiment and to the increased rate of $I_p$ buildup in the simulations.

- The increased $dl_p/dt$ occurs in MHD and two-fluid models.
- Injector voltage is applied more suddenly in the experiment.

The lower frequency (~3 kHz) in the simulations is from evolution of toroidal winding between relaxation events; it is filtered from the lab data.

The higher frequency (~20 kHz) is observed in laboratory and simulation results, and simulations show that it is excited by reconnection events.

[From Eidietis PhD thesis.]
Large-scale flux surfaces form after injection is stopped in the simulations.
**Topology evolution:** Formation of flux-rope rings requires magnetic reconnection.

- Parallel current density in the direction opposite to the injected current is induced as different passes of the flux rope merge.
- The orientation of merging passes is necessarily co-helicity.

Isosurfaces of $\lambda = \mu_0 J_{||} / B$ during an early ring-formation event in the MHD simulation. Red is positive (parallel), and blue is negative (anti-parallel), and frames show intervals that are separated by 30 $\mu$s, starting at 2.63 ms.
Field-line traces reveal how the reconnection process alters magnetic topology.

Traces from a grid of 13 launch points over the injector-source region show single-pass trajectories evolving into multi-pass trajectories as reconnection proceeds. [Left and right images are separated by 5 µs.]
Other flux-rope studies have found ring formation from counter-helicity merging.

- Counter-helicity orientation is favorable for complete reconnection of merging ropes.

Lau and Finn [PoP 3, 3983 (1996)] studied foot-point driven flux ropes and topology evolution. Only the attractive, counter-helicity Case 2 generates a ring.

Co-helicity merging in non-inductive tokamak startup is aided by current drive and radial force-balance.

The ring and driven helical pass slide past each other radially as reconnection proceeds [O'Bryan, PoP 19, 080701 (2012)].

- The ring loses its current drive while the impedance of the driven rope decreases.
- Radial force-balance tends to move the ring inward, as its current decreases, and the driven rope outward.
- The toroidal-equilibrium aspect distinguishes tokamak startup from other flux-rope studies.
- The ‘S’-shaped reconnection current has been noted in other studies. [Intrator, et al., Nature Phys. 5, 521 (2009); Lawrence & Gekelman, PRL 103, 205002 (2009).]
Quasi-separatrix layers (QSLs) have been used to locate reconnection in other aperiodic systems.

- A QSL is a collection of trajectories in a field-line mapping that are extremely sensitive to launch-point position.
- Priest and Démoulin [JGR 100, 23,443 (1995)] use QSLs to identify reconnection in 3D configurations without null points.

Lawrence and Gekelman [PRL 103, 105002 (2009)] identify QSLs in their LAPD data from rope-merging experiments.

Titov, Hornig, and Démoulin [JGR 107, 1164 (2002)] show that reconnection is likely where two QSLs form a hyperbolic flux tube in quadrupole configurations.
QSLs are identified by computing the squashing-degree, $Q$, for field-line traces over the region.

- Tracing magnetic field lines from an entrance surface to an exit surface defines a mapping.
  $$x_1(x_0, y_0) \quad y_1(x_0, y_0)$$
  where $x$ and $y$ are coordinates on the entrance ("0") and exit ("1") surfaces.

- The squashing degree, $Q$, is defined as [Titov, JGR 107, 1164],
  $$Q = J^{-1} \left[ \left( \frac{\partial x_1}{\partial x_0} \right)^2 + \left( \frac{\partial y_1}{\partial x_0} \right)^2 + \left( \frac{\partial x_1}{\partial y_0} \right)^2 + \left( \frac{\partial y_1}{\partial y_0} \right)^2 \right]$$

- With $B_{z1} = B_{z0}$ (both uniform) in our Pegasus computations, $J = 1$.

- We have computed $Q$ for simulation results by tracing from a 2000×60 uniform polar mesh at the bottom of the domain and differencing exit coordinates with respect to entrance coordinates.

- We avoid tracing from $R$ near the central column, however, because $Q$ there is large from the vacuum winding.

$$Q_{vac} (R) = 2 + 4 \Delta \phi^2 (R) \quad \Delta \phi (R) = I \Delta z / R^2 B_z \quad Q_{vac} (R = 5 \text{ cm}) \approx 6 \times 10^6$$
Computations of $Q$ from the Pegasus simulation show evidence of QSLs.

Contours of $\ln(Q)$ for $R_0 > 15$ cm from $t = 2.63$ ms in the MHD simulation plotted at $z = z_0$. Low values are saturated at $\ln(Q) = 8$.

- Large values of $Q$ appear as rolled surfaces when plotted at the entrance plane, $z_0$ ($z = -0.8$ m).

Isosurfaces of $\lambda$ at the same time redisplayed for convenience.
Traces with large $Q$-values cross the site of magnetic reconnection.

- The location of largest negative values of $\lambda$ can be used to find reconnection.
- Overlaying the Poincaré surface for large-$Q$ traces in the corresponding plane shows that the QSL goes through the reconnection site.
- The large-$Q$ traces bifurcate at this site, between the second and third passes of the flux rope ($z \approx -0.2 \text{ m}$), reminiscent of the hyperbolic flux tube description.
- However, they also bifurcate between the first and second passes ($z \approx -0.55 \text{ m}$).

Magnetic punctures for field-lines having $\ln(Q) > 13.2$ and $R_0 > 0.23 \text{ m}$ overlaid on contours of $\lambda$ at $\phi = 3\pi/2$. 
To understand the first bifurcation, we draw an analogy with periodic systems.

- Consider the flux rope like an island chain in a system that is periodic in $z$.
- Choose a wavenumber vector $\mathbf{k}$ to resonate with the vacuum field at $R=R_{inj}$,
  \[ k_\phi = R_{inj}^{-1} \quad k_z = 4 \cdot 2\pi L_{inj}^{-1} \]
- The auxiliary- or helical-$\mathbf{B}$ vanishes at $X$- and $O$-points of an island chain produced by magnetic tearing.
  \[ \mathbf{B_h} = B_R \hat{R} + \mathbf{B} \cdot \hat{k} \hat{k} \]
  - Resonance makes the eq. part $=0$.
  - For tearing ordering with constant-$\psi$,
    \[ \hat{k} \cdot \mathbf{B}_1 = \hat{k} \left[ \left( \partial_\psi / \partial R \hat{R} + i\psi \mathbf{k} \right) \times \mathbf{B}_0 + B_{||} \hat{B}_0 \right] \]
    is also $=0$ at the resonance.
  - $B_R$ from tearing is 0 at $X$- and $O$-points.
- The first bifurcation is analogous to an $X$-point located 1 pass below reconnection.

The same magnetic punctures for $\phi = 3\pi/2$ overlaid on contours of $|B_h|$ computed with the simulation data and periodic-analogy $k$. 
Results on field-line length as a function of initial position show evidence of chaotic scattering.

- We have recorded field-line length for 50,000 traces launched from the line segment $10 \text{ cm} \leq R_0 \leq 50 \text{ cm}$, $\phi_0 = -\pi/4$, $z_0 = -80 \text{ cm}$.
- Regions with sharply increased length, repeating at finer scales, indicates chaotic scattering [Lau and Finn, PoP 3, 3983 (1996)].
Poloidal flux generation: Net poloidal flux accumulates over a sequence of ring formation events.

Contour lines of toroidally averaged poloidal flux superposed on contours of $\lambda$ when $I_p = 26$ kA.

Contours of poloidal flux and $\lambda$ when $I_p = 42$ kA.

- Significant poloidal flux amplification (relative to vacuum) occurs after reversal of large-scale poloidal field [O’Bryan, PoP 19, 080701 (2012)].
Our helicity source induces net poloidal flux directly, but dynamo-like activity affects the global distribution.

- Our source has $\oint E_{\text{inj}} \cdot \hat{\phi} R d\phi \neq 0$, so we are not modeling all of the current-drive processes in the experiment.

- The directly induced flux in the simulation is localized to the source region.

- MHD $-\langle \tilde{V} \times \tilde{B} \rangle$ and Hall $(ne)^{-1} \langle \tilde{J} \times \tilde{B} \rangle$ dynamo effects from the correlation of fluctuations can redistribute poloidal flux.

- Poynting’s theorem can be applied to the toroidally averaged magnetic field. For low-frequency dynamics:

$$\frac{1}{2\mu_0 \partial t} \left( \langle B \rangle^2 \right)_{\partial t} + \frac{1}{\mu_0} \nabla \cdot \left[ \langle E \rangle \times \langle B \rangle \right] = -\langle E \rangle \cdot \langle J \rangle$$

- Energy density in the mean field increases where $\langle E \rangle \cdot \langle J \rangle < 0$.

- With $\delta_i \approx 10$ cm and $\rho_s \approx 3$ cm at or below the simulated flux-rope diameter, and $\delta_{\eta} \approx 2$ cm, two-fluid effects are expected to be important.
The MHD simulation shows poloidal flux concentration and $\langle E \rangle \cdot \langle J \rangle < 0$ where a ring forms.

Contour lines of poloidal flux overlaid on contours of emf from $-\langle \mathbf{v} \times \mathbf{b} \rangle$ during the early ring formation event. 

Contours of toroidally symmetric $J_\phi$. 
Essentially all of the emf at the ring is from the rope’s rotation into a horizontal plane.

Contributions to emf from -<v×b> separated by component during ring formation.

- Toroidally asymmetric vertical motion rotates a pass of the helical rope into a constant-z plane.
- The cause of the MHD dynamo effect is easy to understand in this case.
The two-fluid simulation shows shaping associated with reconnection from the Hall dynamo effect.

- The MHD dynamo effect supplies power to the ring.
- The Hall dynamo effect acts on a scale that is smaller than the ring.

Contour lines of $\langle J_\phi \rangle$ overlaid on contours of emf from $-\langle \mathbf{v} \times \mathbf{b} \rangle$ during a ring formation event in the two-fluid simulation.

Contour lines of $\lambda$ overlaid on contours of emf from $\langle \mathbf{j} \times \mathbf{b} \rangle$.
Conclusions

• Simulated current-filament evolution in non-inductive Pegasus startup shows properties of flux-rope dynamics noted in solar physics and basic plasma studies.

• Ring formation from co-helicity merging may be unique for tokamak startup configurations due to toroidal equilibrium considerations.

• Quasi-separatrix layers observed during ring formation are consistent with published descriptions of 3D reconnection.
  • They pass through and bifurcate in the region where reconnection occurs.
  • They also show bifurcation at a non-reconnecting $X$-point-like location, which is another separatrix-like property.
  • MHD dynamo effect is evident in ring formation and is associated with rotation of a helical ring into a horizontal plane.
  • Whether flux ropes survive late in the experiment’s current injection will be determined by new diagnostics.

See UW-CPTC 13-06 for a summary of the flux-rope analysis.
Cartoon physics of Faraday’s and Ampere’s laws helps explain how $E_{\text{inj}}$ induces current.

Apply localized $E_{\text{inj}} = -E$

$\vec{B}_0$

$\vec{E}$

$\vec{B}_0 + \vec{B}$

[\(\vec{B}\) adds a little twist like a candy wrapper.]

$\vec{J} \times \vec{B}_0$ launches a torsional Alfvén wave that tends to unwrap and spread localized twist.

End View

Side View

With Damping

$\vec{J} \times \vec{B}_0 = F$