Nonlinear Extended MHD Simulation for Magnetic Plasma Confinement

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As the development of magnetic plasma confinement approaches conditions for ignition, ...

Alcator C-mod, Massachusetts Institute of Technology

TFTR, Princeton Plasma Physics Laboratory

Joint European Torus, European Fusion Development Agreement

DIII-D, General Atomics Corporation
... the need for predictive simulation increases.

Critical ‘macroscopic’ topics include:
1. Internal kink stability
2. Neoclassical tearing excitation and control
3. Edge localized mode control
4. Wall-mode feedback

[2002 Snowmass Fusion Summer Study]

planned International Thermonuclear Experimental Reactor (ITER)
- Fusion power: 500 MW
- Stored thermal energy: 10s of MJ
Outline

• Introduction
• Macroscopic plasma dynamics
  • Characteristics
  • Simulation examples
• PDE system
• Computational modeling
  • Numerical methods
    • High-order spatial representation
    • Time-advance for drift effects
• Implementation
• Conclusions
Macroscopic Plasma Dynamics

- Magnetohydrodynamic (MHD) or MHD-like activity limits operation or affects performance in all magnetically confined configurations.
- Analytical theory has taught us which physical effects are important and how they can be described mathematically.
- Understanding consequences in experiments (and predicting future experiments) requires numerical simulation:
  - Sensitivity to equilibrium profiles and geometry
  - Strong nonlinear effects
  - Competition among physical effects
Fusion plasmas exhibit enormous ranges of temporal and spatial scales.

- Nonlinear MHD-like behavior couples many of the time- & length-scales.
- Even within the context of resistive MHD modeling, there is stiffness and anisotropy in the system of equations.
Examples of Nonlinear Macroscopic Simulation

1) MHD evolution of the tokamak internal kink mode (m=1, n=1)
   • Plasma core is exchanged with cooler surrounding plasma.

   M3D simulation of NSTX [W. Park]

   Evolution of pressure and magnetic topology from a NIMROD simulation of DIII-D
Examples of Nonlinear Macroscopic Simulation (continued)

2) Disruption (Loss of Confinement) events

- Understanding and mitigation are critical for a device the size of ITER.

Simulated event in DIII-D starts from an MHD equilibrium fitted to laboratory data.

Resulting magnetic topology allows parallel heat flow to the wall.

simulation & graphics by S. Kruger and A. Sanderson
3) Helical island formation from tearing modes
- Being weaker instabilities, tearing modes are heavily influenced by non-MHD effects.
- Tearing modes are usually non-disruptive but lead to significant performance loss.
- Slow evolution makes the system very stiff.
Examples of Nonlinear Macroscopic Simulation (continued)

4) Edge localized modes

- Strong gradients at the open/closed flux boundary drive localized modes.
- Heat transport to the wall occurs in periodic events—can be damaging if not controlled.

Medium-wavenumber modes are unstable.

Nonlinear coupling in MHD simulation leads to localized structures that are suggestive of bursty transport.

Simulation by D. Brennan

MHD description is insufficient, however.
PDE System: a fluid-based description

Evolution equations:

\[
m_i n \left( \frac{\partial V}{\partial t} + V \cdot \nabla V \right) = J \times B - \nabla p - \nabla \cdot \Pi_i(V)
\]

\[
\frac{\partial n}{\partial t} + \nabla \cdot (V n) = 0
\]

\[
\frac{3n}{2} \left( \frac{\partial T_\alpha}{\partial t} + V_\alpha \cdot \nabla T_\alpha \right) = -n T_\alpha \nabla \cdot V_\alpha - \nabla \cdot q_\alpha + Q_\alpha \quad \alpha = i, e
\]

\[
\frac{\partial B}{\partial t} = -\nabla \times \left[ \frac{1}{ne} (J \times B - \nabla p_e) - V \times B + \eta J \right]
\]

\[
\mu_0 J = \nabla \times B
\]

Closure relations (for collisional conditions):

\[
\Pi_{gv} = \frac{m_i p_i}{4 e B} \left[ \hat{b} \times W \cdot (I + 3\hat{b}\hat{b}) - (I + 3\hat{b}\hat{b}) \cdot W \times \hat{b} \right], \quad \left( W \equiv \nabla V + \nabla V^T - \frac{2}{3} I \nabla \cdot V \right)
\]

\[
\Pi_{\parallel} = \frac{p_i \tau_i}{2} \left( \hat{b} \cdot W \cdot \hat{b} \right) (I - 3\hat{b}\hat{b})
\]

\[
q_i = -n \left[ \chi_{\parallel i} \hat{b} \hat{b} + \chi_{\perp i} (I - \hat{b}\hat{b}) \right] \cdot \nabla T_i + 2.5 p_i (eB)^{-1} \hat{b} \times \nabla T_i
\]

\[
q_e = -n \left[ \chi_{\parallel e} \hat{b} \hat{b} + \chi_{\perp e} (I - \hat{b}\hat{b}) \right] \cdot \nabla T_e - 2.5 p_e (eB)^{-1} \hat{b} \times \nabla T_e
\]
PDE System (continued)

Fluid models of macroscopic MHD activity in MFE plasmas are characterized by extreme stiffness and anisotropy.

- **Stiffness**: Time-scales that impact nonlinear MHD evolution include
  - Parallel particle motion leading to parallel thermal equilibration over flux surfaces in 100s of nanoseconds to microseconds.
  - MHD wave propagation over global scales in microseconds.
  - Magnetic fluctuations and tearing in hundreds of microseconds to milliseconds.
  - Nonlinear profile modification and transport in tens to hundreds of milliseconds.
  - Global resistive diffusion over seconds.

- **Anisotropy**: Magnetization of nearly collisionless particles leads to
  - Effective thermal diffusivity ratios, $\chi_\parallel/\chi_\perp$, reaching and exceeding $10^{10}$.
  - Shear wave resonance that allows nearly singular behavior of MHD modes.

Both properties make the numerical solution of fluid models challenging when applied to MFE conditions, and they must be addressed when developing an algorithm for studying fusion MHD.
Computational Modeling

Challenges:
• Anisotropy relative to the strong magnetic field
  • Distinct shear and compressive behavior
  • Extremely anisotropic heat flow
• Magnetic divergence constraint
• Stiffness arising from multiple time-scales
• Weak resistive dissipation

Helpful considerations:
• Linear effects impose the time-scale separation
• Typically free of shocks
Modeling: Spatial Representation

• The NIMROD code ([http://nimrodteam.org](http://nimrodteam.org) and JCP 195, 355, 2004) uses finite elements to represent the poloidal plane and finite Fourier series for the periodic direction.
• Polynomial basis functions may be Lagrange or Gauss-Lobatto-Legendre. Degree>1 provides
  • High-order convergence without uniform meshing
  • Curved isoparametric mappings

Packed mesh for DIII-D ELM study

SSPX

LDX
Modeling: Spatial Representation (continued)

- Polynomials of degree>1 also provide
  - **Control of magnetic divergence error**

‘Error diffusion’ is added to Faraday’s law:

\[
\frac{\partial B}{\partial t} = -\nabla \times E + \kappa_{\text{div}b} \nabla \cdot B
\]

\[
\int dx \left\{ c^* \cdot \Delta b + g \Delta t \kappa_{\text{div}b} (\nabla \cdot c^*) (\nabla \cdot \Delta b) \right\}
\]

\[
= \Delta t \int dx \left\{ \kappa_{\text{div}b} (\nabla \cdot c^*) (\nabla \cdot b) - (\nabla \times c^*) \cdot E \right\}
\]

\[
- \Delta t \int ds \times E \cdot c^*
\]

for all vector test functions \(c^*\).

The ratio of DOF/constraints is 3 in the limit of large polynomial degree.

Scalings show convergence rates expected for first derivatives.
• Polynomials of degree>1 also provide
  • Resolution of extreme
    anisotropies (Lorentz force
    and diffusion)

Simple 2D test:

• Homogeneous Dirichlet boundary
  conditions on $T$

• Heat and perpendicular current have
  sources. $2\pi^2 \cos(\pi x) \cos(\pi y)$

• Analytically, the solution is
  independent of $\chi_\parallel$,

$$T(x, y) = \chi^{-1}_\perp \cos(\pi x) \cos(\pi y)$$

• The resulting $T^{-1}(0,0)$ measures the
effective $\chi_\perp$, including the numerical
truncation error.
Reproducing Transport with Magnetic Islands

With anisotropy, heat transport across magnetic islands is a competition between parallel and perpendicular processes.

Critical island width vs. $\chi_{||}/\chi_{\perp}$

$W_c$ shows where diffusion time-scales match [Fitzpatrick, PoP 2, 825 (1995)].
Modeling: Time-advance algorithms

- Stiffness from fast parallel transport and wave propagation requires implicit algorithms.
- Semi-implicit methods for MHD have been refined over the last two decades (DEBS, XTOR, NIMROD).
- The underlying scheme is leapfrog,

\[
\begin{align*}
V^j & \quad V^{j+1} \\
\vdots & \quad \vdots \\
t^j & \quad t^{j+1/2} \quad t^{j+1} \quad t^{j+3/2} \\
B^{j+1/2} & \quad B^{j+3/2} \\
n^{j+1/2}, T^{j+1/2} & \quad n^{j+3/2}, T^{j+3/2}
\end{align*}
\]

with the following implicit operator in the velocity advance to stabilize waves without numerical dissipation.

\[
L(\Delta V) = \frac{1}{\mu_0} \{ \nabla \times [\nabla \times (\Delta V \times B_0)] \} \times B_0 + J_0 \times \nabla \times (\Delta V \times B_0) + \nabla (\Delta V \cdot \nabla p_0 + \gamma p_0 \nabla \cdot \Delta V)
\]
Modeling: Time-advance algorithms (continued)

- A new implicit leapfrog advance is being implemented for modeling two-fluid effects (drifts and dispersive waves).

\[
m_{i}n^{j+1/2} \left( \frac{\Delta V}{\Delta t} + \frac{1}{2} V^{j} \cdot \nabla \Delta V + \frac{1}{2} \Delta V \cdot \nabla V^{j} \right) - \Delta t L^{j+1/2}(\Delta V) + \nabla \cdot \Pi_{i}(\Delta V) = J^{j+1/2} \times B^{j+1/2}
\]

\[
- m_{i}n^{j+1/2} V^{j} \cdot \nabla V^{j} - \nabla p^{j+1/2} - \nabla \cdot \Pi_{i}(V^{j})
\]

\[
\frac{\Delta n}{\Delta t} + \frac{1}{2} V^{j+1} \cdot \nabla \Delta n = -\nabla \cdot \left( V^{j+1} \cdot n^{j+1/2} \right)
\]

\[
\frac{3n}{2} \left( \frac{\Delta T_{\alpha}}{\Delta t} + \frac{1}{2} V^{j+1}_{\alpha} \cdot \nabla \Delta T_{\alpha} \right) + \frac{1}{2} \nabla \cdot q_{\alpha}(\Delta T_{\alpha}) = -\frac{3n}{2} V^{j+1}_{\alpha} \cdot \nabla T^{j+1/2}_{\alpha} - nT^{j+1/2}_{\alpha} \nabla \cdot V^{j+1}_{\alpha}
\]

\[
- \nabla \cdot q_{\alpha}(T^{j+1/2}_{\alpha}) + Q^{j+1/2}_{\alpha}
\]

\[
\frac{\Delta B}{\Delta t} + \frac{1}{2} V^{j+1} \cdot \nabla \Delta B + \frac{1}{2} \nabla \times \frac{1}{ne} \left( J^{j+1/2} \times \Delta B + \Delta J \times B^{j+1/2} \right) + \frac{1}{2} \nabla \times \eta \Delta J
\]

\[
= -\nabla \times \left[ \frac{1}{ne} \left( J^{j+1/2} \times B^{j+1/2} - \nabla p_{e} \right) - V^{j+1} \times B^{j+1/2} + \eta J^{j+1/2} \right]
\]

- A fully time-centered approach is also possible.
Analysis shows the leapfrog with implicit magnetic advance to be numerically stable and accurate on two-fluid waves.

\[ \theta = 0.04\pi \quad \Delta t = 1, \quad c_s^2/\nu_A^2 = 0.1. \quad \theta = 0.46\pi \]
Modeling: Implementation

- Algebraic systems from 2D and 3D operations are solved during each time-step (~10,000s over a nonlinear simulation).
- 3D systems result from nonlinear fluctuations in toroidal angle.
  - Toroidal couplings are computed with FFTs in matrix-free solves.
- 2D systems represent coupling over the FE mesh only, and matrix elements are computed.
  - They are also generated for preconditioning the matrix-free solves.

Example sparsity pattern for a small mesh of biquartic elements—after static condensation but before reordering.
Modeling: Implementation (continued)

- Solving algebraic systems is the dominant performance issue.
- Iterative methods scale well but tend to perform poorly on ill-conditioned systems.
- Collaborations with TOPS Center researchers Kaushik and Li led us to parallel direct methods with reordering → SuperLU (http://crd.lbl.gov/~xiaoye/SuperLU/).

SuperLU DIST

SuperLU improves NIMROD performance by a factor of 5 in nonlinear simulations.
Conclusions

The challenges of macroscopic plasma modeling are being met by developments in numerical and computational techniques, as well as advances in hardware.

• High-order spatial representation controls magnetic divergence error and allows resolution of anisotropies that were previously considered beyond reach.

• SciDAC-fostered collaborations have resulted in huge performance gains through sparse parallel direct solves (with SuperLU).

• Algorithm development is proceeding for two-fluid modeling. Help with matrix-free non-Hermitian solves is welcome!