Capabilities and Highlights of the NIMROD MHD Code

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Outline

• Introduction
• Example and problem characteristics
• Physical models
• Spheromak ⇒ accretion disk applications
• Summary
The project has been a multi-institutional effort since 1996.

<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
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Presently, there is non-team-member use of the NIMROD code at LLNL, IFS, Univ. of WA, UCLA, and AIST-Japan.
Goals for NIMROD
(Non-Ideal Magnetohydrodynamics with Rotation, an Open Discussion Project)

• Develop a simulation code package for studying three-dimensional, nonlinear electromagnetic activity in laboratory fusion experiments.

• Allow flexibility in the geometry and physics models used in simulations.

• Allow efficient computation on a wide range of platforms from PCs to massively parallel supercomputers.

• Provide user-friendly features, such as a graphical interface and documentation, and make the code publicly available. [http://nimrodteam.org]

• Apply techniques such as integrated product development and quality function deployment in design and development.
A nonlinear simulation of a classical tearing-mode illustrates the primary challenges for nonlinear fusion MHD.

**NIMROD Simulation**
- \( S = \tau_R / \tau_A = 10^6 \)
- \( Pm = \tau_R / \tau_v = 0.1 \)
- \( \tau_A = 1 \mu s \)
- \( \beta << 1\% \) to avoid GGJ stabilization

**DIII-D L-mode Startup Plasma**
[R. LaHaye, Snowmass Report]
- \( S = 1.6 \times 10^6 \)
- \( Pm = 4.5 \)
- \( \tau_A = 0.34 \mu s \)
- \( \tau_E = 0.03 s \)
STIFFNESS: small $\Delta'$ (linear $\gamma\tau_A=5\times10^{-4}$) leads to nonlinear evolution over the energy confinement time-scale.

Magnetic Island Width vs. Time

- 20,000 semi-implicit time-steps evolve solution for times $> \tau_E$.
- Explicit computation is impossible $\rightarrow 2\times10^8$ time-steps.
ANISOTROPY: transport properties in the parallel and perpendicular directions, relative to the three-dimensional magnetic field configuration, are vastly different.

**Critical Island Width vs. $\chi_{||}/\chi_{\text{perp}}$**

- $W_c$ shows where diffusion time-scales match [Fitzpatrick, PoP 2, 825 (1995)].
- Numerical results measure width needed to affect $T$-profile.

- 5th-order accurate biquartic finite elements resolve anisotropies.
Furthermore, we want to study nonlinear MHD in a variety of configurations.

NIMROD uses 2D finite elements (that are general with respect to the degree of polynomials used for basis functions) for the poloidal plane and finite Fourier series for the periodic direction, which may be toroidal, azimuthal, or a periodic linear coordinate.
Physical models for *macroscopic* dynamics are based on fluid-like magnetohydrodynamic (MHD) descriptions.

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} + \kappa_{\text{divb}} \nabla \cdot \mathbf{B}
\]

\[
\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J}
\]

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B}
\]

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot D \nabla n
\]

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \nu \rho \nabla \mathbf{V}
\]

\[
\frac{n}{\gamma - 1} \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -p \nabla \cdot \mathbf{V} + \nabla \cdot n \left( (\chi_{\parallel} - \chi_{\perp}) \hat{\mathbf{b}} \hat{\mathbf{b}} + \chi_{\perp} \mathbf{I} \right) \cdot \nabla T + Q
\]

- Density and magnetic-divergence diffusion are for numerical purposes.
- Present two-fluid modeling is limited, but the formulation is being revised.
For detailed modeling of nonlocal kinetic effects, we have developed an approach that solves drift kinetic equations to determine closures. [Held, Callen, Hegna, Sovinec, PoP 8, 1171 (2001)]

- Drift kinetic equations are solved allowing for maximal ordering of collisional and free-streaming terms (no assumptions on $\nu$ wrt $|v_\parallel \cdot \nabla|$)
- Kinetic fluxes are determined via integration along characteristics in a 3D solution.
  - Nonlocal fluxes interact self-consistently with fluid moments.
  - A semi-implicit advance allows large time-step.

Simulation particles (~PIC) are being added for ion kinetic effects.
Despite the challenges of strong global coupling and complicated data structures (for geometric flexibility), we have been able to show scaling to large numbers of processors. [Communication via MPI]

- This nonlinear problem has a 64x128 mesh of biquartic finite elements and 6 Fourier components.
- Here, scaling and efficiency fall-off at around 400 processors.
- Kinetic physics packages, implemented by E. Held and C. Kim, are more readily decomposed for parallel computing.
A study of spheromak helicity injection and sustainment found flux amplification as a result of pinch current instability.

- Steady 2D solution to the resistive MHD equations is a pinch that is unstable to $n=1$.
- Full 3D evolution finds the conversion of toroidal flux to poloidal flux as part of the MHD saturation ($S=10^3-10^4$).
- Simply connected geometry is required.
- Geometric flexibility also allows study of gun-driven configurations.
Flux amplification is a direct result of the resistive $n=1$ pinch mode and the azimuthal average of the resulting 3D field. [Sovinec, Finn, del-Castillo-Negrete, PoP 8, 475 (2001).]

- Formation includes magnetic reconnection.
- With sufficient drive, the final state exhibits limit-cycle behavior.
- Removing drive allows flux-surface formation as perturbations decay preferentially, and toroidal current is driven by induction.
- Detailed studies of confinement during transients are in progress at LLNL and Univ. of WI.
Feasibility Tests of Accretion Disk Simulations

Mesh and poloidal flux contours from vacuum and two subsequent times.

- Mesh spacing must increase with radius.
- A dipole flux distribution expands as differential rotation is applied at the disk (z=0).
- Future (3D) numerical work will use NIMROD to explore flux conversion in the astrophysical configuration.
Summary

• NIMROD has been developed to address MHD problems with extreme stiffness and anisotropy.
• Its geometric flexibility will allow application to astrophysical problems of interest.
• Development of more advanced physical models continues under OFES and SCIDAC support.